CHAPTER 5.1 CONCRETE DESIGN THEORY

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5.1.1 INTRODUCTION

Reinforced concrete is a predominant material in California highway structures, especially after the wide adoption of prestressing technology in the 1950s. Reinforced concrete bridges, prestressed or non-prestressed, account for about 90% of all bridges in the California highway system today. Some advantages of reinforced concrete are:

- Fluidity allows job-site casting into structural and aesthetic shapes
- Lower cost when compared to off-site component fabrication
- Fire resistance and weather durability reduce maintenance cost

The concrete design has evolved from Allowable Stress Design (ASD), or Working Stress Design (WSD), to Ultimate Strength Design (USD) or Load Factor Design (LFD), to today's Limit State Design (LSD) or Load and Resistance Factor Design (LRFD). Since adopted in 1994, the AASHTO LRFD Bridge Design Specifications is in its 8th Edition (AASHTO, 2017). While not perfect, the new methods in LRFD are more rational than the LFD methodology and entail an amount of effort appropriate with the support of current theories and methods compared to those available when the LFD was developed. Changes include:

- Unified design provisions for reinforced and prestressed concrete
- Modified compression field theory for shear and torsion
- Alternative Strut and Tie modeling techniques for shear and flexure
- End zone analysis for tendon anchorages
- New provisions for segmental construction
- Improved provisions for estimating prestress losses

Chapter 5.1 will summarize the general aspects of concrete component design using the *AASHTO LRFD Bridge Design Specifications, 8th Edition* (AASHTO, 2017) with the *California Amendments* (Caltrans, 2019a), i.e, AASHTO-CA BDS-8, while Chapter 5.2 will give a detailed description of the design procedure for post-tensioned box girder bridges, Chapters 5.3 to 5.5 will cover the design of precast prestressed girder bridges, Chapters 5.6 and 5.7 will address concrete bent caps and concrete columns, respectively. The design of the concrete deck is covered in Chapter 9.1. Equations and Articles referenced in this document are based on AASHTO unless noted otherwise.

5.1.2 STRUCTURAL MATERIALS

5.1.2.1 Concrete

An important property of concrete is the compressive strength. Concrete with 28-day compressive strength f_c'= 3.6 ksi is commonly used in conventionally reinforced concrete structures while concrete with higher strength is used in prestressed concrete structures. The California Amendments (CA) Article 5.4.2.1 (Caltrans, 2019a) specifies the minimum design strength of 3.6 ksi for reinforced concrete and requires the minimum design strength of 4.0 ksi for prestressed concrete and decks. When a higher strength is needed for a project, designers should consider various factors including the cost and local material availability.

5.1.2.2 Reinforcing Steel

Steel reinforcing bars are manufactured as plain or deformed bars. Per Caltrans Standard Specifications, Section 52-1.02B (Caltrans 2018), the main reinforcing bars are generally deformed. Reinforcing bars must be low-alloy steel deformed bars conforming to requirements in ASTM A706/A706M with a 60 ksi yield strength, except that deformed or plain billet-steel bars conforming to the requirements on ASTM A615/A615M, Grade 40 or 60, may be used as reinforcement in some minor structures as specified in Caltrans Standard Specifications. Welded wire reinforcement may be used as a substitution in certain cases.

5.1.2.3 Prestressing Steel

Two types of high-tensile strength steel used for prestressing steel can be found in Caltrans Standard Specifications, Section 50-1.02B:

Strands: ASTM A416/A416M Grade 270, low relaxation, uncoated
ASTM A882/A882M Grade 270, low relaxation, epoxy coated

Bars: ASTM A722/A722MType II

Strands and bars may be grouted or un-bonded. Multiple-wire strands yield in the 250 to 270 ksi range, bars begins the idealized plastic elongation between 75 and 160 ksi. Modern beams, slabs, and ground anchor designs use 0.375 in., 0.5 in. or 0.6 in. diameter strands. Historically, stress relieved strands have been replaced with low-relaxation material to allow a better estimation of losses and increased initial jacking forces.

5.1.3 DESIGN LIMIT STATES

Concrete bridge components are designed to satisfy the requirements of service, strength, and extreme-event limit states for load combinations specified in AASHTO Table 3.4.1-1 (AASHTO, 2017) with Caltrans Amendments (Caltrans, 2019a). The following are the four limit states into which the load combinations are grouped:

1) *Service Limit States*

   Concrete stresses, deformations, cracking, distribution of reinforcement, deflection, and camber are investigated at service limit states.

   Service I: Normal loading combined with wind loads, and earth pressures.

   Service III: Crack control and tension in the prestressed concrete

   Service IV: Crack control in post-tensioned precast column sections

2) *Fatigue Limit States*

Fatigue of the reinforcement need not be checked for fully prestressed concrete members satisfying requirements of the service limit state. Fatigue need not be investigated for concrete deck slabs in multi-girder application, approach slabs, slab bridges, or reinforced-concrete box culverts. For fatigue requirements, refer to CA (5.5.3), (3.6.1.4)

3) *Strength Limit States*

Axial, flexural, shear strength, and stability of concrete components are investigated at strength limit states. Resistance factors are based on CA (5.5.4.2)

Strength I: Basic load (HL-93) without wind

Strength II: Owner specified load (Permit)

Strength III: Wind on structure

Strength IV: Structure with high DL/LL (>7)

Strength V: Wind on both structure and live load

4) *Extreme Event Limit States*

Concrete bridge components and connections must resist extreme event loads due to earthquake and collision forces, but not simultaneously.

5.1.4 FLEXURAL DESIGN

5.1.4.1 Strength Limit States

5.1.4.1.1 Design Requirements

In flexural design, the basic strength design requirement can be expressed as follows:

$$M_u \leq \phi M_n = M_r \qquad (5.1.4.1\text{-}1)$$

where M_u is the factored moment at the section (kip-in.); M_n is the nominal flexural resistance (kip-in.); and M_r is the factored flexural resistance of a section in bending (kip-in.) as defined in Article (5.6.3.2.1-1).

In assessing the nominal resistance for flexure, the provisions unify the strength design of conventionally reinforced and prestressed concrete sections based on their behavior at the ultimate limit state. In earlier LFD Specifications, a flexure member was designed so that the section would fail in a tension-controlled mode, thus there was a maximum reinforcement ratio. In the current AASHTO, there is no explicit upper bound for reinforcement. There is a distinction between compression and tension-controlled section based on the strain in the extreme tension steel. To compensate for the less ductile behavior of compression-controlled sections, a lower value of resistance reduction factor ϕ is assigned to "compression-controlled" sections compared to "tension-controlled" sections. The LRFD procedure defines a transition behavior region in which the resistance factor ϕ, to be used for strength computation, varies linearly with the strain in the extreme steel fibers. The design of sections falling in this behavior region may involve an iterative procedure.

Here are a few terms used to describe the flexural behavior of the reinforced section:

Balanced strain condition: Strain in extreme tension steel reaches its yielding strain as the concrete in compression reaches its assumed ultimate strain of 0.003.

Compression-controlled section: When the net tensile strain in the extreme tensile steel (NTS) from a linear strain distribution is less than or equal to the compression-controlled strain limit (ε_{cl}) just as the concrete in compression reaches its assumed strain limit of 0.003, a brittle failure condition is expected and the section behaves more like a column than a beam. Article (5.6.4) discusses reinforcement limits, slenderness, and basic confinement for compression-controlled sections.

$\phi = 0.75$

Tension-controlled section: When NTS is greater or equal to the Tension-controlled strain limit (ε_{tl}) just as the concrete in compression reaches its assumed strain limit of 0.003, a ductile failure condition is expected and the section behaves more like a beam. ε_{tl} is dependent upon the tensile strength of the reinforcement, see Articles (C5.6.2.1), (5.6.3) discuss flexural resistance, minimum reinforcement, moment redistributions, and deflections.

Resistance factors are as follows:

$\phi = 1.0$ for precast prestressed sections

$\phi = 0.95$ for cast-in-place prestressed sections

$\phi = 0.90$ for non-prestressed sections

Transition region: Compression controlled strain limit (ε_{cl}) < NTS < Tension-controlled strain limit (ε_{tl}). For the transition region, the resistance factor is calculated using linear interpolation.

Figure 5.1.4-1 illustrates the three regions and equations for resistance factors for flexural resistance per CA (5.5.4.2), Figure C5.5.4.2-1.

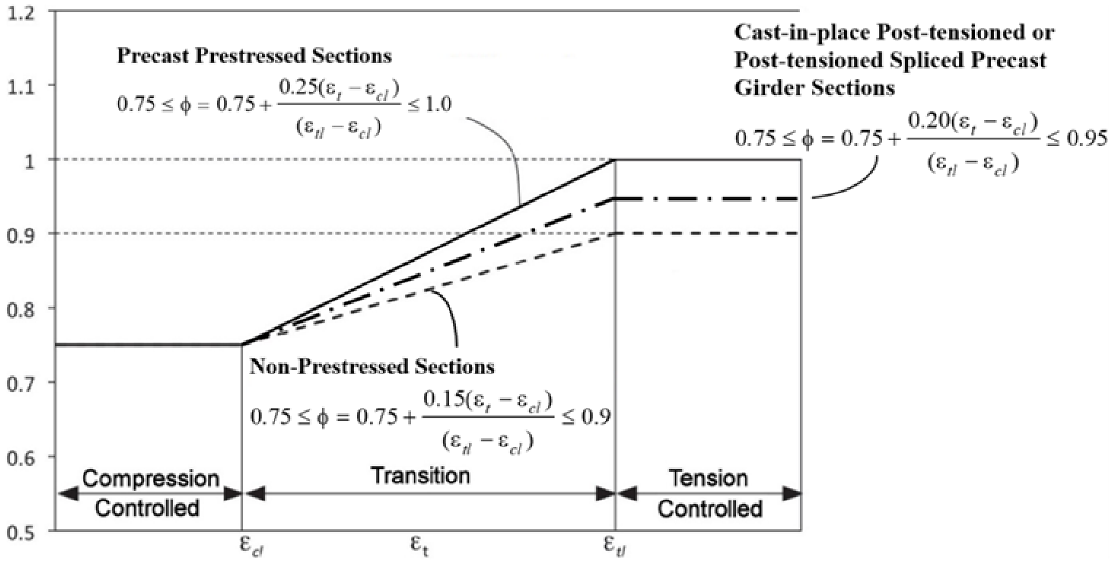


Figure 5.1.4-1 Resistance Factor Variation

5.1.4.1.2 Nominal Flexural Resistance

The provisions for conventionally reinforced and prestressed concrete are now one-and-the-same. The basic assumptions used for flexural resistance as specified in Article 5.6.2.1 are as follows:

- Plane section remains plane after bending, i.e., strain is linearly proportional to the distance from the neutral axis, except for deep members, corbels and brackets see AASHTO (5.6.2.1).
- For unconfined concrete, maximum usable strain at the extreme concrete compression fiber is not greater than 0.003. For confined concrete, a maximum usable strain exceeding 0.003 may be used if verified.
- Stress in the reinforcement is based on its stress-strain curve except for D-regions.
- Tensile strength of concrete is neglected.
  - Concrete compressive stress-strain distribution is assumed to be rectangular, parabolic, or any shape that results in predicted strength in substantial agreement with the test results. An equivalent rectangular compression stress block of $\alpha_1 f_c'$ over a zone bounded by the edges of the cross-section and a straight line located parallel to the neutral axis at the distance $a = \beta_1 c$ from the extreme compression fiber may be used in lieu of a more exact concrete stress distribution, where c is the distance measured perpendicular to the neutral axis. The stress block factor α_1 shall be taken as 0.85 for design compressive strength of concrete not exceeding 10.0 ksi. For design compressive strengths of concrete exceeding 10.0 ksi, α_1 shall be reduced at a rate of 0.02 for each 1.0 ksi of strength in excess of 10.0 ksi, except that α_1 shall not be taken to be less than 0.75.

For a T-beam section, there are two cases (Figure 5.1.4-2) depending on where the neutral axis falls into:

- Case 1: flanged section when the neutral axis falls into the web
- Case 2: rectangular section when the neutral axis falls into the flange

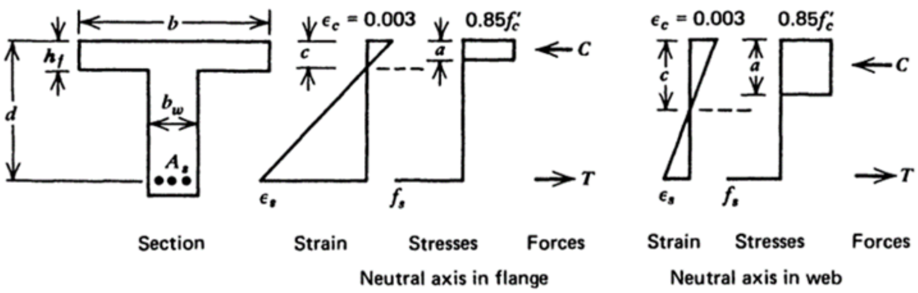


Figure 5.1.4-2 Stress and Strain Distribution of T-Beam Section in Flexure (shown with mild reinforcement only)

For flanged sections, the M_n can be calculated by the following equation assuming the compression flange depth is less than $a = \beta_1 c$:

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right) - A'_s f'_s\left(d'_s - \frac{a}{2}\right) + \alpha_1 f'_c\left(b - b_w\right) h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)$$

(AASHTO 5.6.3.2.2-1)

where α_1 is the stress block factor; a is the depth of equivalent rectangular stress block (in.); c is the distance from the extreme compression fiber to the neutral axis (in.); b is the width of the compression face of the member (in.); b_w is the web width (in.); h_f is the thickness of flange (in.); d_s is the distance from the compression face to the centroid of mild tensile reinforcement (in.); d'_s is the distance from the compression face to the centroid of mild compression reinforcement (in.); and d_p is the distance from the compression face to the centroid of prestressing steel (in.); A_s is the area of mild tensile reinforcement (in.^{2}) and A_{ps} is the area of prestressing steel (in.^{2}); A'_s is the area of mild compressive reinforcement (in.^{2}); f_s is the stress in mild tensile steel (ksi); f'_s is the stress in the mild steel compression reinforcement (ksi) and f_{ps} is the stress in prestressing steel (ksi).

For rectangular sections, let $b_w = b$. The last term of the above equation will be dropped.

For circular and other nonstandard cross-sections, strain-compatibility must be used.

To evaluate the prestressing stresses, the following equations can be used:

For bonded tendons:

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$ (AASHTO 5.6.3.1.1-1)

in which:

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right)$$ (AASHTO 5.6.3.1.1-2)

For flanged sections:

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A_s^{'} f_s^{'} - \alpha_1 f_c^{'}\left(b - b_w\right)h_f}{\alpha_1 f_c^{'}\beta_1 b_w + kA_{ps}\dfrac{f_{pu}}{d_p}}$$ (AASHTO 5.6.3.1.1-3)

For rectangular sections:

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A_s^{'} f_s^{'}}{\alpha_1 f_c^{'}\beta_1 b + kA_{ps}\dfrac{f_{pu}}{d_p}}$$ (AASHTO 5.6.3.1.1-4)

For unbonded tendons:

$$f_{ps} = f_{pe} + 900\left(\frac{d_p - c}{l_e}\right) \le f_{py}$$ (AASHTO 5.6.3.1.2-1)

in which:

$$l_e = \frac{2l_i}{2 + N_s}$$ (AASHTO 5.6.3.1.2-2)

For flanged sections:

$$c = \frac{A_{ps}f_{ps} + A_s f_s - A_s^{'} f_s^{'} - \alpha_1 f_c^{'}\left(b - b_w\right)h_f}{\alpha_1 f_c^{'}\beta_1 b_w}$$ (AASHTO 5.6.3.1.2-3)

For rectangular sections:

$$c = \frac{A_{ps}f_{ps} + A_s f_s - A_s^{'} f_s^{'}}{\alpha_1 f_c^{'}\beta_1 b}$$ (AASHTO 5.6.3.1.2-4)

where f_{py} and f_{pu} are the yield and ultimate tensile strength of prestressing steel respectively; f_{pe} is the effective stress in prestressing steel after loss (ksi); l_e is the effective tendon length (in.); l_i is the length between anchorages (in.); and N_s is the number of support hinges crossed by the tendon between anchorages.

5.1.4.1.3 Reinforcement Limits

As mentioned before, there is no explicit limit on the maximum reinforcement for tensile sections in flexure. Sections can be over reinforced but shall be compensated for reduced ductility in the form of a reduced resistance reduction factor.

The minimum reinforcement shall be provided so that, M_r, is at least equal to the lesser of the cracking moment and 1.33 M_u per AASHTO (5.6.3.3)

5.1.4.2 Service Limit States

Service limit states are used to satisfy stress limits, deflection, and cracking requirements. To calculate the stress and deflection, the designer can assume concrete behaves elastically. The modulus of elasticity can be evaluated according to the code specified formula such as Article (5.4.2.4). The reinforcement and prestressing steel are usually transformed into concrete. For normal weight concrete with w_c = 0.145 kcf and design compression strength up to 10.0 ksi, the modulus of elasticity, may be taken from either of the following:

$$E_c = 33{,}000 K_1 w_c^{1.5} \sqrt{f'_c} \qquad \text{(AASHTO C5.4.2.4-2)}$$

$$E_c = 1{,}820 \sqrt{f'_c} \qquad \text{(AASHTO C5.4.2.4-3)}$$

where w_c is the unit weight of concrete (kcf); K_1 is a correction factor for the source of aggregates to be taken as 1.0 unless determined by the physical test, and as approved by Caltrans.

For prestressed concrete members, the prestressing force and the concrete strength are determined by meeting stress limits in the service limit states, and then checked in the strength limit states for ultimate capacity. All other members are designed in accordance with the requirements of strength limit states first. The cracking requirement is satisfied by proper reinforcement distribution, see Section 5.1.4.3.

5.1.4.2.1 Stress Limits for Concrete

To design the prestressed members, the following stress limits as specified in AASHTO (5.9.2.3), AASHTO Tables 5.9.2.3.2a-1 and 5.9.2.3.1b-1, CA Table 5.9.2.3.2b-1. They are summarized in Table 5.1.4.2-1.

Table 5.1.4.2- 1 Stress Limits for Concrete

Condition	Stress	Location	Stress Limits
Temporary Stress before loss	Tensile	In area other than Precompressed Tensile Zone and without bonded tendons or reinforcement	$0.0948\lambda\sqrt{f'_{ci}} \leq 0.2$ (ksi)
		In area with bonded tendons or reinforcement sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of $0.5f_y$, not to exceed 30 ksi	$0.24\lambda\sqrt{f'_{ci}}$
	Compression	All locations	$0.65f'_{ci}$
Final Stress after loss at service load	Tensile	In the Precompressed Tensile Zone, assuming uncracked section:	
		• Components with bonded tendons or reinforcement that are subjected to not worse than moderate corrosion condition, and are in Caltrans "non-freeze-thaw area"	$0.19\lambda\sqrt{f'_c} \leq 0.6$ (ksi)
		• Components with bonded tendons or reinforcement that are subjected to severe corrosive conditions, or are in Caltrans "Freeze-thaw area"	$0.0948\lambda\sqrt{f'_c} \leq 0.3$ (*ksi*)
		• Components with unbonded tendons	No tension
	Compression	All locations due to:	
		• Sum of Permanent loads and effective prestress	$0.45f'_c$
		• All other load combinations	$0.6\phi_w f'_c$
Permanent loads only	Tensile	Precompressed Tensile Zone with bonded prestressing tendons or reinforcement	No tension

5.1.4.2.2 Control of Cracking

Cracks occur in concrete components due to:

- Loading conditions
- Thermal effects
- Deformations

Cracks occur whenever tension stress in the member exceeds the modulus of rupture of concrete. The severity of flexural cracking in a concrete member can be controlled by providing optimized tension reinforcement layouts, using smaller bar sizes, and providing tighter spacing.

Per Article 5.6.7, the spacing, *s*, of mild steel reinforcement in the layer closest to the tension face is given by:

$$s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c \qquad \text{(AASHTO 5.6.7-1)}$$

where d_c is= the thickness of the concrete cover measured from extreme tension fiber to center of the closest flexural reinforcement (in.); γ_e is the exposure factor= 1.00 for Class 1 exposure condition, = 0.75 for Class 2 exposure condition

$$\beta_s = 1 + \frac{d_c}{0.7\left(h - d_c\right)} \qquad \text{(AASHTO 5.6.7-2)}$$

where *h* is the overall thickness or depth of the component (in.); f_{ss} is the calculated tensile stress in steel reinforcement at the service limit state not to exceed 0.6 f_y (ksi)

> *Note:* In the above equation, the spacing, *s*, of the bar reinforcing steel is inversely proportional to the stress in the reinforcing steel.

Also, per CA Article 5.6.7 (Caltrans 2019a):

- Class 1 exposure condition applies when cracks can be tolerated due to reduced concerns for appearance and/or corrosion.
- Class 2 exposure condition applies when there are increased concerns for appearance and/or corrosion (for example, in areas where de-icing salts are used). Class 2 exposure condition applies to all bridge decks.

5.1.4.2.3 Moment of Inertia of Cracked Reinforced Concrete Sections

This section provides general calculation procedures for the moment of inertia of cracked reinforced rectangular and flanged concrete sections, I_{cr}. The moment of inertia are mainly used to calculate deflections (Article 5.6.3.5.2), stresses in nonprestressed reinforcement at the service limit state (Article 5.6.7), and the fatigue limit state (Article 5.5.3.2) as specified in AASHTO-CA BDS-8 (AASHTO, 2017; Caltrans, 2019).

Basic Assumptions

The following assumptions should be used in the moment of inertia calculations:

- Steel reinforcement area is transferred to an equivalent concrete area by multiplying the steel area by a modular ratio, $n = E_s/E_c$, (Article 5.6.1). E_s is modulus of elasticity of steel (ksi); and E_c is modulus of elasticity of concrete (ksi).
- The contribution of concrete in the tensile zone is fully neglected.

Neutral axial Location

For a rectangular or a flanged section, the neutral axial location can be determined by the following:

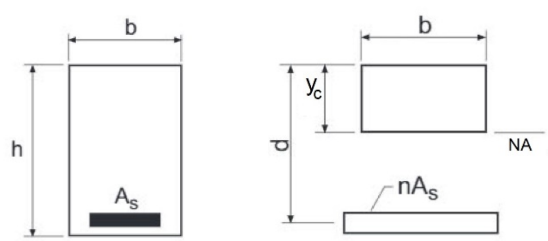
$$y_c = \sqrt{B^2 + C} - B \tag{5.1.4.2-1}$$

where y_c is the depth of the compression zone defined as the distance between the extreme compression fiber and the NA (in.); B, C are transformed section factors depending on the section dimensions and reinforcement.

Rectangular Sections

Singly Reinforced Sections

A typical singly reinforced rectangular section is shown in Figure 5.1.4.2-1.



(a) Gross Section (b) Cracked Section

Figure 5.1.4.2-1 Singly Reinforced Rectangular Section

$$B = \frac{nA_s}{b} \tag{5.1.4.2-2}$$

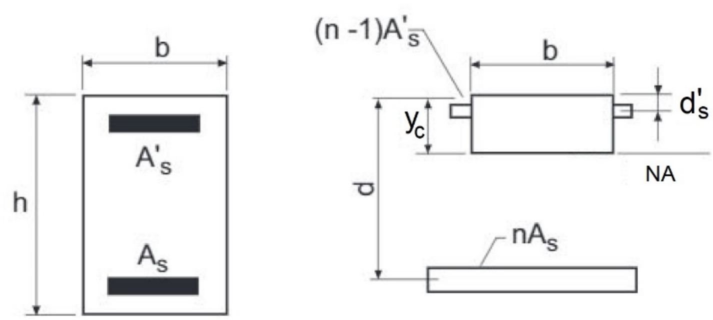
$$C = \frac{2ndA_s}{b} \tag{5.1.4.2-3}$$

$$I_{cr} = \frac{1}{3} b y_c^3 + nA_s (d - y_c)^2 \quad (5.1.4.2\text{-}4)$$

where A_s is the area of nonprestressed tension reinforcement (in.^{2}); b is the width of the compression face of the section (in.) ; d is the effective depth of the section defined as the distance between the extreme compression fiber and the centroid of the tension longitudinal reinforcement (in.); h is the depth of the section (in.); I_{cr} is the moment of Inertia of the cracked section, transformed to concrete (in.^{4}); NA is the neutral axis

Doubly Reinforced Sections

A typical doubly reinforced rectangular section is shown in Figure 5.1.4.2.2.



(b) Gross Section (b) Cracked Section

Figure 5.1.4.2-2 Doubly Reinforced Rectangular Section

$$B = \frac{nA_s + (n-1)A_s'}{b} \quad (5.1.4.2\text{-}5)$$

$$C = \frac{2ndA_s + (n-1)d_s'A_s'}{b} \quad (5.1.4.2\text{-}6)$$

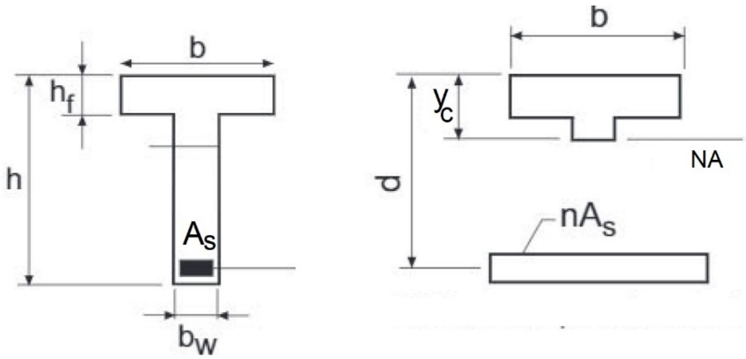
$$I_{cr} = \frac{1}{3} b y_c^3 + nA_s (d - y_c)^2 + (n-1)A_s' (y_c - d_s')^2 \quad (5.1.4.2\text{-}7)$$

where A_s' is the area of nonprestressed compression reinforcement (in.^{2}); d_s' is the distance from the extreme compression fiber to the centroid compression reinforcement (in.).

Flanged Sections

Singly Reinforced Sections

A typical singly reinforced flanged section is shown in Figure 5.1.4.2-3.



(c) Gross Section (b) Cracked Section

Figure 5.1.4.2-3 Singly Reinforced Flanged Section

$$B = \frac{h_f\left(b - b_w\right) + nA_s}{b_w} \tag{5.1.4.2-8}$$

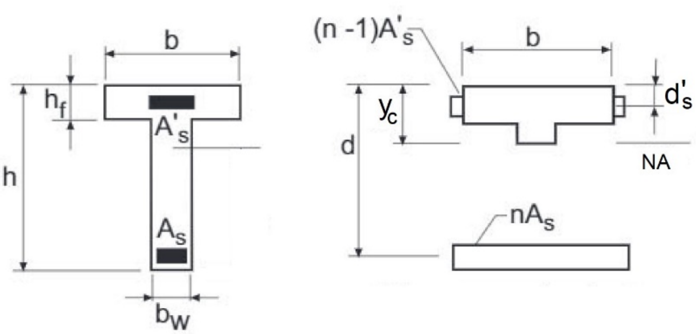
$$C = \frac{2\left[h_f^2\left(b - b_w\right)/2 + ndA_s\right]}{b_w} \tag{5.1.4.2-9}$$

$$I_{cr} = \frac{b\,y_c^3}{3} - \frac{\left(b - b_w\right)\left(y_c - h_f\right)^3}{3} + nA_s\left(d - y_c\right)^2 \tag{5.1.4.2-10}$$

where b is the effective width of the flange in compression (in.); b_w is the web width (in.); h_f is the compression flange depth of an I- or T-section (in.)

Doubly Reinforced Sections

A typical doubly reinforced flanged section is shown in Figure 5.1.4.2-4.



(d) Gross Section (b) Cracked Section

Figure 5.1.4.2-4 Doubly Reinforced Flanged Section

$$B = \frac{h_f\left(b-b_w\right)+nA_s+\left(n-1\right)A_s'}{b_w} \tag{5.1.4.2-11}$$

$$C = \frac{2\left[h_f^2\left(b-b_w\right)/2+ndA_s+\left(n-1\right)d_s'A_s'\right]}{b_w} \tag{5.1.4.2-12}$$

$$I_{cr} = \frac{b\,y_c^3}{3}-\frac{\left(b-b_w\right)\left(y_c-h_f\right)^3}{3}+nA_s\left(d-y_c\right)^2+\left(n-1\right)A_s'\left(y_c-d_s'\right)^2 \tag{5.1.4.2-13}$$

5.1.4.3 Fatigue Limit States

As per CA Article 5.5.3.1 and Article 5.5.3.2, the stress range in reinforcing bars due to the fatigue load combination should be checked and should satisfy:

$$\gamma(\Delta f) \le (\Delta F)_{TH} \quad \text{(AASHTO 5.5.3.1-1)}$$

$$\left(\Delta F\right)_{TH} = 26-\frac{22f_{min}}{f_y} \quad \text{(AASHTO 5.5.3.2-1)}$$

Where γ is the load factor (see below); (Δf) is the live load stress range (ksi); $(\Delta F)_{TH}$ is the constant-amplitude fatigue threshold (ksi); and f_{min} is the minimum live load stress resulting from the Fatigue I load combination, combined with the more severe stress from either the unfactored permanent loads or the unfactored permanent loads, shrinkage, and creep-induced external loads, positive if tension, negative if compression (ksi)

For the fatigue check:

- The fatigue load combination is given in the CA Table 3.4.1-1. A load factor (γ) of 1.75 is specified on the live load (Fatigue truck) for Fatigue I load combination.
- A Fatigue truck is one design truck with a constant 30-ft spacing between the 32.0-kip axles as specified in CA (3.6.1.4).
- Apply the *IM* = 1.15 factor to the fatigue loads.
- Check both top and bottom reinforcement to ensure that the stress range in the reinforcement under the fatigue load stays within the range specified in the above equation.

5.1.5 SHEAR DESIGN

5.1.5.1 Basic Concept of Modified Compression Field Theory

Perhaps the most significant change for the concrete design in *AASHTO LRFD Bridge Design Specifications* is the shear design methodology. It provides two methods: The Sectional Method, and the Strut and Tie Method. Both methods are acceptable to Caltrans. The Sectional Method, which is based on the Modified Compression Field Theory (MCFT), provides a unified approach for shear design for both prestressed and reinforced concrete components. For a detailed derivation of this method, please refer to the book by Collins and Mitchell (1991).

The two approaches are summarized as follows:

- Sectional Method
  - Plane section remains plane – Basic Beam Theory
  - Based on Modified Compression Field Theory (MCFT)
  - Used for most girder designs, except disturbed-end, or D-Regions
  - Used for undisturbed, or B-Regions

- Strut and Tie Method
  - Plane section does not remain plane
  - Used in “disturbed regions”, including deep beams, brackets, and corbels
  - Examples of usage: Design of Bent Caps (clear span to depth ratio less than 4); pile caps; anchorage zones (general or local); area around openings

In this discussion, only the Sectional Method will be outlined.

Compression Field Theory (CFT) is highlighted as follows:

- Angle for compressive strut (or crack angle) is variable
- Plane section remains plane (for strain compatibility)
- Tension in concrete is ignored
- Strains incorporate the effects of axial forces, shear, and flexure
- Methodology utilizes average stress and strain in reinforcement
- The shear capacity is related to the compression in diagonally cracked concrete through equilibrium

This theory is further modified by including the strength of concrete in tension and is then referred to as the Modified Compression Field Theory (MCFT).

5.1.5.2 Shear Strength

The factored shear resistance, V_r, is given by:

$$V_r = \phi V_n \quad \text{(CA 5.7.2.1-1)}$$

And the factored shear V_u shall be less than or equal to that of the factored shear resistance V_r.

$$V_u \leq V_r \quad \text{(CA 5.7.2.1-1a)}$$

According to Article 5.7.3.3, the nominal shear resistance, V_n, shall be determined as:

$$V_n = V_c + V_s + V_p \quad \text{(AASHTO 5.7.3.3-1)}$$

But, the total resistance provided by concrete and steel: $V_c + V_s$ should not exceed 0.25 $f'_c b_v d_v$. In the end region of the beam-type element when it is not built integrally with the support, $V_c + V_s$ should not exceed $0.18 f'_c b_v d_v$, if the shear stress in the beam exceeds this value, the region shall be designed using the Strut and Tie Method and special consideration should be given to detailing.

$$V_c = 0.0316\beta\lambda\sqrt{f'_c}\, b_v d_v \quad \text{(AASHTO 5.7.3.3-3)}$$

$$V_s = \frac{A_v f_y d_v \left(\cot\theta + \cot\alpha\right)\sin\alpha}{s} \quad \text{(AASHTO 5.7.3.3-4)}$$

where V_p is the component in the direction of applied shear of the prestressing force (kip); b_v is the effective web width (in.); and d_v is the effective shear depth (in.); θ is the angle of inclination of diagonal compressive stress (degree); α is the angle of inclination of transverse reinforcement to the longitudinal axis (degree); β is the factor indicating the ability of diagonally cracked concrete to transmitting tension and shear; s, is the spacing of transverse reinforcement; A_v is the area of transverse reinforcement within a distance of s (in^2) ; λ is the concrete density modification factor (Article 5.4.2.8)

Determine the effective shear depth, d_v. As specified in Article 5.7.2.8, the effective shear depth, d_v, is taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure. It needs not be taken to be less than the greater of $0.9d_e$ or $0.72h$ where h is the overall thickness or depth of a member.

In which:

$$d_e = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} \quad \text{(AASHTO 5.7.2.8-2)}$$

Per Article C5.7.2.3, transverse reinforcement must be provided in all regions where there is a significant chance of diagonal cracking. Transverse reinforcement must be provided where:

$$V_u > 0.5\phi\,(V_c + V_p) \qquad \text{(AASHTO 5.7.2.3-1)}$$

5.1.5.2.1 Method 1: Simplified Procedure for Nonprestressed Sections

CA Article C5.7.3.4 allows the simplified procedure for shear design for nonprestressed sections (Caltrans 2019a). For concrete footings in which the distance from the point of zero shear to the face of the column, pier, or wall is less than $3d_v$, and for other nonprestressed concrete sections not subjected to axial tension and containing at least the minimum amount of transverse reinforcement or having an overall depth of less than 16.0 in., assume $\theta = 45^o$ and $\beta = 2.0$ in the general procedure outlined in Article 5.7.3.4.1.

5.1.5.2.2 Method 2: The Iterative Procedure using Tabularized Values

For members not covered by the above article, CA Article C5.7.3.4 states that the following iterative procedure shall be used for shear design. For sections containing at least the minimum transverse reinforcement, β and θ values calculated from the MCFT are given as functions of ε_x, shear stress v_u, and f'_c in AASHTO Table B5.2-1. ε_x is taken as the calculated longitudinal strain at mid-depth of the member when the section is subjected to M_u, N_u, and V_u.

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right)}{2\left(E_sA_s + E_pA_{ps}\right)} \le 0.001 \qquad \text{(AASHTO B5.2-3)}$$

For sections containing less than the minimum transverse reinforcement, β and θ values calculated from the MCFT are presented as functions of ε_x, and the crack spacing parameter s_{xe} in AASHTO Table B5.2-2. ε_x is taken as the largest calculated longitudinal strain which occurs within the web of the member when the section is subjected to M_u, N_u, and V_u.

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right)}{\left(E_sA_s + E_pA_{ps}\right)} \le 0.002 \qquad \text{(AASHTO B5.2-4)}$$

If the value of ε_x from AASHTO Equations (B5.2-3) or (B5.2-4) is negative, the strain is taken as:

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right)}{2(E_cA_{ct} + E_sA_s + E_pA_{ps})} \quad \text{(AASHTO B5.2-5)}$$

The crack spacing parameter s_{xe}, is determined as:

$$s_{xe} = s_x \frac{1.38}{a_g + 0.63} \leq 80 \text{ in.} \quad \text{(AASHTO B5.2-6)}$$

Where s_x is the crack spacing parameter; a_g is the maximum aggregate size (in.)

As one can see, ε_x, β, and θ are all inter-dependent. So, design is an iterative process:

1. Calculate shear stress demand v_u at a section and determine the shear ratio (v_u/f'_c)
2. Calculate ε_x at the section based on normal force (including p/s), shear, and bending and an assumed value of θ
3. Longitudinal strain ε_x is the average strain at mid-depth of the cross section
4. Knowing v_u/f'_c & ε_x, obtain the values of β and θ from the table
5. Recalculate ε_x based on the revised value of θ; repeat iteration until convergence in θ is achieved.

where v_u, the shear stress on the concrete, should be determined as:

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v} \quad \text{(AASHTO 5.7.2.8-1)}$$

5.1.5.2.3 Flexure – Shear Interaction

The effect of shear forces on the longitudinal reinforcement is determined and the adequacy of the reinforcement is checked using the AASHTO Eq. 5.7.3.5-1. Longitudinal reinforcement along with the vertical steel stirrups and the compression strut in concrete constitute the truss mechanism that carries the applied loads.

The check of the adequacy of the longitudinal reinforcement may result in the added length of and/or the added amount of longitudinal reinforcement. Figure 5.1.5-1 shows the concept of direct/indirect loading and support, along with the demand on the longitudinal steel from flexure (solid line) and shear (dashed line).

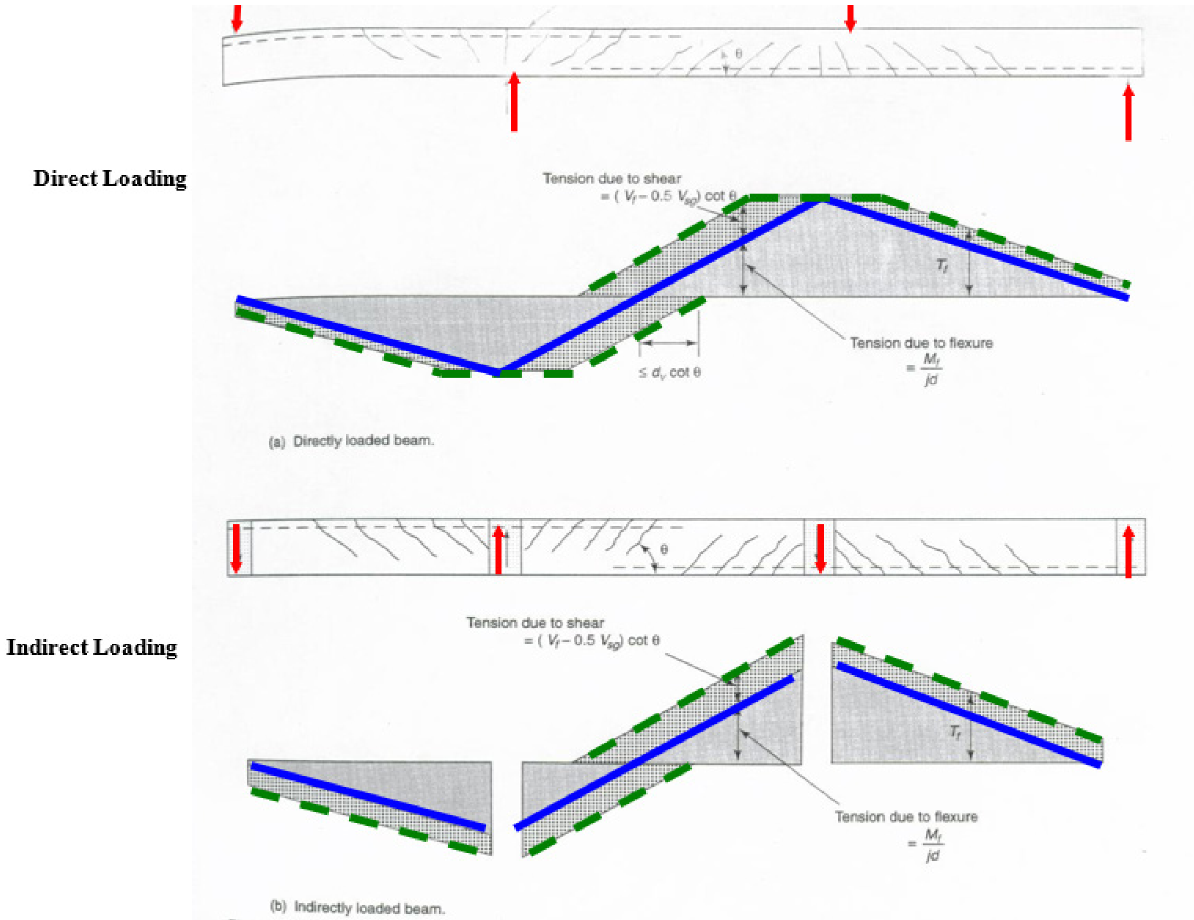


Figure 5.1.5-1 Direct and Indirect Loading/Support

As can be seen from Figure 5.1.5-1, the amount of longitudinal steel need not exceed the maximum amount due to flexure demands in the case of direct loading/support. The additional shear demands on the longitudinal steel can be overcome by extending the length of the longitudinal bar reinforcement. However, in the case of girders framed onto other girders at equal depth or height (indirect loading), the shear demand is likely to result in an additional amount of longitudinal steel beyond what is needed to meet the maximum flexure demands.

A direct loading/support case is typical for drop bent cap beams. Precast and steel girders are applying the load atop the bent cap. Columns are also directly supporting the cap. Since Caltrans practices no curtailment of the longitudinal reinforcement in bent caps (due to the nature of seismic loading), there is no need to check for shear-flexure interaction. Indirect loadings/supports are often encountered in box girder construction. For a box girder bridge, where the girders are framing into the integral bent cap, the amount of longitudinal reinforcement in the girder needs to be checked using the shear-flexure interaction equation. Similarly, for an integral bent cap constructed integrally with columns, the longitudinal reinforcement in the bent cap shall be checked for shear-flexure interaction.

At every location, the following three possible load conditions shall be checked:

- Maximum shears and associated moments
- Maximum positive moments and associated shears
- Maximum negative moments and associated shears

The longitudinal steel must satisfy:

$$A_{ps}f_{ps} + A_s f_y \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left| \frac{V_u}{\phi_v} - V_p \right| - 0.5V_s \right)\cot\theta \quad \text{(AASHTO 5.7.3.5-1)}$$

Where ϕ_f is the resistance factor for moment; ϕ_c is the resistance factor for axial load,; ϕ_v is the resistance factor for shear.

Note: V_s shall not be taken more than $\dfrac{V_u}{\phi_v}$.

5.1.5.2.4 Transverse Reinforcement Limits

- Minimum Transverse Reinforcement

  Except for segmental post-tensioned concrete box girder bridges, the area of steel should satisfy:

$$A_v \geq 0.0316\sqrt{f'_c}\,\frac{b_v s}{f_y} \quad \text{(AASHTO 5.7.2.5-1)}$$

- Maximum Spacing of Transverse Reinforcement

  The spacing of the transverse reinforcement should not exceed the maximum spacing, s_{max}, determined as:

- If $v_u < 0.125\,f'_c$ then:

  $s_{max} = 0.8d_v \leq 18.0$ in. (CA 5.7.2.6-1)

- If $v_u \geq 0.125\,f'_c$ then:

  $s_{max} = 0.4d_v \leq 12.0$ in. (AASHTO 5.7.2.6-2)

5.1.6 COMPRESSION DESIGN

As stated previously, when a member is subjected to a combined moment and the compression force, its resulting strain can be in a compression-controlled state. The compression design procedure applies. The following effects are considered in addition to bending: degree of end fixity; member length; variable moment of inertia; deflections; and duration of loads. This chapter will only cover the two basic cases: pure compression, and combined flexure and compression ignoring slenderness. Article 5.6.4.3 provides an approximate method for evaluating slenderness effects.

5.1.6.1 Factored Axial Compression Resistance – Pure Compression

The factored axial resistance of concrete compressive members, symmetrical about both principal axes, is taken as:

$$P_r = \phi P_n \qquad \text{(AASHTO 5.6.4.4.-1)}$$

In which: P_n , the nominal compression resistance, can be evaluated for the following two cases:

For members with spiral reinforcement:

$$P_n = 0.85[k_c f'_c (A_g - A_{st} - A_{ps}) + f_y A_{st} - A_{ps} (f_{pe} - E_p \varepsilon_{cu})] \qquad \text{(AASHTO 5.6.4.4-2)}$$

For members with tie reinforcement:

$$P_n = 0.80[k_c f'_c (A_g - A_{st} - A_{ps}) + f_y A_{st} - A_{ps} (f_{pe} - E_p \varepsilon_{cu})] \qquad \text{(AASHTO 5.6.4.4-3)}$$

where k_c is 0.85 for the design compressive strength not exceeding 10.0 ksi. For the design compressive strength exceeding 10.0 ksi, k_c shall be reduced at a rate of 0.02 for each 1.0 ksi of strength in excess of 10.0 ksi, except that k_c shall not be less than 0.75; A_g is the gross area of the section (in.^{2}); A_{st} is the total area of longitudinal mild reinforcement (in.^{2}); A_{ps} is the area of prestressing steel (in.^{2}); E_p is the modulus of elasticity of prestressing steel (ksi); and ε_{cu} is the failure strain of concrete in compression.

In order to achieve the above resistance, the ratio of spiral or hoop reinforcement to the total concrete core, $\rho_s = 4A_{sp}/(d_c S)$ shall satisfy the following:

$$\rho_s \geq 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{yh}} \qquad \text{(AASHTO 5.6.4.6-1)}$$

where f_{yh} is the specified yield strength of transverse reinforcement (ksi); A_c is the area of column core measured to the outside of the hoops (in^2).

To achieve more ductility for seismic resistance, Caltrans has its own set of requirements for spirals, hoops, and ties. For further information, please refer to the current version of the Caltrans Seismic Design Criteria (Caltrans, 2019b).

5.1.6.2 Combined Flexure and Compression

When a member is subjected to a compression force, end moments are often induced by eccentric loads. The end moments rarely act solely along the principal axis. So, at any given section for analyzing or design, the member is normally subjected to biaxial bending as well as compression. Furthermore, to analyze or design a compression member in a bridge substructure, many load cases need to be considered.

Under special circumstances, AASHTO allows designers to use an approximate method to evaluate biaxial bending combined with axial load (Article 5.6.4.5). Generally, designers rely on computer programs based on equilibrium and strain compatibility to generate a moment-axial interaction diagram. For cases like noncircular members with biaxial flexure, an interaction surface is required to describe the behavior. Figure 5.1.6-1 shows a typical moment-axial load interaction surface for a concrete section (Park and Pauley 1975).

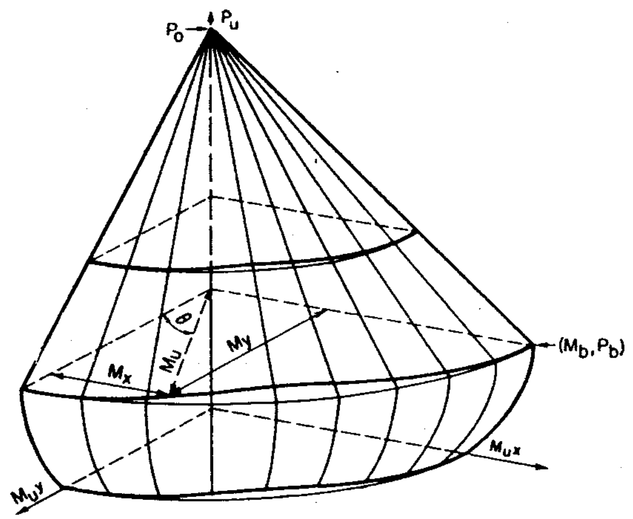


Figure 5.1.6-1 Moment-Axial Interaction Surface of a Noncircular Section

For bridge column design, the entire surface has little value to designers. Rather, the design program normally reports a series of lines, basically slices of the surface, at spaced intervals, such as 15º. Figure 5.1.6-2 is an example plot from a computer program.

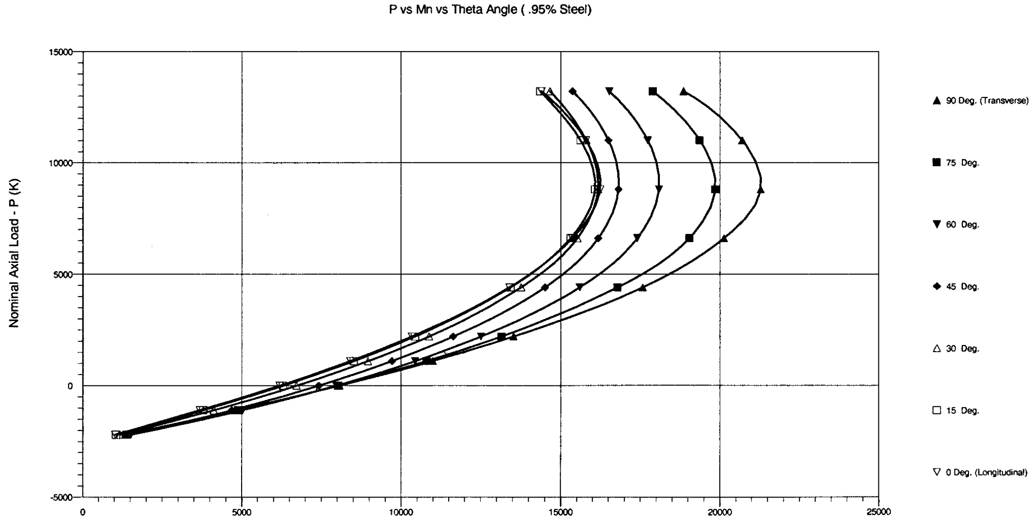


Figure 5.1.6-2 Interaction Diagrams generated by a computer program

From these lines, it can be seen that below the balanced condition (the point corresponding to the maximum moment in the interaction diagram) the moment capacity increases with an increase in axial load. Therefore, when designing a column, it is incorrect to simply take a set of maximum axial load with maximum bending moments. The following combinations need to be evaluated:

1. $M_{ux\ max}$, associated M_{uy} and P_u
2. $M_{uy\ max}$, associated M_{ux} and P_u
3. $P_{u\ max}$ and associated M_{ux} and M_{uy}

Special Notes:

- Detailed column design is covered in Chapter 5.7.
- For load cases 1 and 2, γ_p, the permanent load factor, corresponding to the minimum axial forces shall be used.
- P_n and M_n shall be multiplied by ϕ, a resistance factor specified in Article CA 5.5.4.2 (Caltrans 2019a).
- Slenderness effect shall be evaluated with an appropriate nonlinear analysis program or the use of approximate methods shown in Article 5.6.4.3.
- In California, the column design is normally controlled by seismic requirements. That topic is not covered in this chapter.

5.1.6.3 Reinforcement Limits

The maximum area of prestressed and non-prestressed longitudinal reinforcement for non-composite compression members is as follows:

$$\frac{A_s}{A_g} + \frac{A_{ps} f_{pu}}{A_g f_y} \leq 0.08 \qquad \text{(AASHTO 5.6.4.2-1)}$$

and

$$\frac{A_{ps} f_{pe}}{A_g f'_c} \leq 0.30 \qquad \text{(AASHTO 5.6.4.2-2)}$$

The minimum area of prestressed and non-prestressed longitudinal reinforcement for non-composite compression members is as follows:

$$\frac{A_s}{A_g} + \frac{A_{ps} f_{pu}}{A_g f_y} \geq 0.135 \frac{f'_c}{f_y} \qquad \text{(AASHTO 5.6.4.2-3)}$$

For design compressive strength of normal weight concrete up to 15.0 ksi where the unfactored permanent loads do not exceed $0.4 A_g f'_c$, the reinforcement ratio need not be greater than

$$\frac{A_s}{A_g} + \frac{A_{ps} f_{pu}}{A_g f_y} \leq 0.015 \qquad \text{(AASHTO 5.6.4.2-4)}$$

Due to seismic concerns, Caltrans put further limits on longitudinal steel in columns. For such limits, please refer to the latest version of the *Caltrans Seismic Design Criteria* (Caltrans, 2019b).

NOTATION

A_c = area of core measured to the outside diameter of the spiral (in.^{2})

A_g = gross area of section (in.^{2})

A_{ps} = area of prestressing steel (in.^{2})

A_s = area of non-prestressed tension steel (in.^{2})

A'_s = area of compression reinforcement (in.^{2})

A_{st} = total area of longitudinal mild steel reinforcement (in.^{2})

A_v = area of transverse reinforcement within distance s (in.^{2})

a = depth of equivalent rectangular stress block (in.)

B = transformed section factors depending on the section dimensions and reinforcement

b = width of compression face of the member; effective width of the flange in compression (in.)

b_v = effective web width taken as the minimum web width (in.)

b_w = web width (in.)

C = transformed section factors depending on the section dimensions and reinforcement

c = distance from the extreme compression fiber to the neutral axis (in.)

d = distance from compression face to centroid of tension reinforcement (in.)

d_e = effective depth from extreme compression fiber to the centroid of tensile force in the tensile reinforcement (in.)

d_p = distance from extreme compression fiber to centroid of prestressing strand (in.)

d_s = distance from extreme compression fiber to centroid of non-prestressed tensile reinforcement (in.)

d'_s = distance from the extreme compression fiber to the centroid compression reinforcement (in.)

d_v = effective shear depth (in.)

E_c = modulus of elasticity of concrete (ksi)

E_p = modulus of elasticity of prestressing tendons (ksi)

E_s = modulus of elasticity of reinforcing bars (ksi)

f'_c = specified compressive strength of concrete (ksi)

f_{pe} = effective stress in prestressing steel after losses (ksi)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance of members is required (ksi)

f_{pu} = specified tensile strength of prestressing steel (ksi)

f_{py} = yield strength of prestressing steel (ksi)

f_s = stress in mild tensile reinforcement at nominal flexural resistance (ksi)

f'_s = stress in mild compression reinforcement at nominal flexural resistance (ksi)

f_y = specified minimum yield strength of reinforcing bars (ksi)

f_{yh} = specified yield strength of transverse reinforcement (ksi)

h = depth of the section (in.);

h_f = thickness of flange (in.)

l_e = effective tendon length (in.)

I_{cr} = moment of inertia of the cracked section, transformed to concrete (in.^{4})

l_i = length of tendon between anchorages (in.)

M_{cr} = cracking moment (kip-in.)

M_n = nominal flexural resistance (kip-in.)

M_r = factored flexural resistance of a section in bending (kip-in.)

M_u = factored moment at the section (kip-in.)

M_{ux} = factored moment at the section in respect to the principal x axis (kip-in.)

M_{uy} = factored moment at the section in respect to the principal y axis (kip-in.)

NA = neutral axis

N_u = factored axial force (kip)

N_s = number of support hinges crossed by the tendon between anchorages or discretely bonded points

n = modular ratio

P_n = nominal axial resistance of a section (kip)

P_o = nominal axial resistance of a section at zero eccentricity (kip)

P_r = factored axial resistance of a section (kip)

P_u = factored axial load of a section (kip)

s = spacing of reinforcing bars (in.)

V_c = nominal shear resistance provided by tensile stresses in the concrete (kip)

V_n = nominal shear resistance of the section considered (kip)

V_p = component in the direction of the applied shear of the effective prestressing forces; positive if resisting the applied shear (kip)

V_r = factored shear resistance (kip)

V_s = shear resistance provided by the shear reinforcement (kip)

V_u = factored shear force (kip)

v_u = average factored shear stress on the concrete (ksi)

y_c = depth of the compression zone defined as the distance between the extreme compression fiber and the NA (in.)

α = angle of inclination of transverse reinforcement to the longitudinal axis (º)

β = factor relating effect of longitudinal strain on the shear capacity of concrete, as indicated by the ability of diagonally cracked concrete to transmit tension

β_1 = ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone

γ = load factor

ε_{cu} = failure strain of concrete in compression (in./in.)

ε_x = Longitudinal strain in the web reinforcement on the flexural tension side of the member (in./in.)

θ = angle of inclination of diagonal compressive stress (º)

ϕ = resistance factor

ϕ_c = resistance factor for compression

ϕ_f = resistance factor for moment

ϕ_v = resistance factor for shear

ϕ_w = hollow column reduction factor

ρ_s = ratio of spiral reinforcement to the total volume of column core

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