CHAPTER 5.5 PRECAST PRETENSIONED VOIDED SLABS

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5.5.1 INTRODUCTION

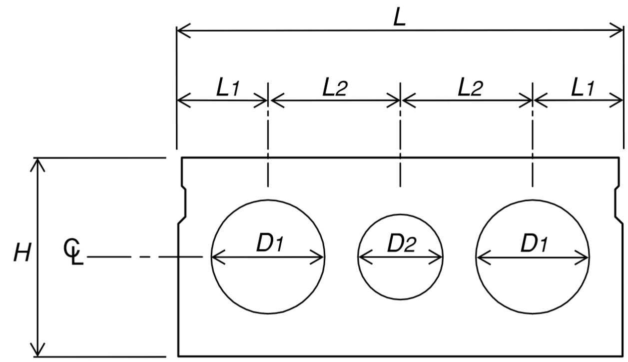
Precast pretensioned solid and voided slabs, referred to herein as voided slabs, are typically used for relatively short span structures with limited or no clearance for falsework. Placed side-by-side, voided slabs can serve as the driving surface, which makes them ideal for Accelerated Bridge Construction (ABC) applications. However, grinding or an overlay consisting of reinforced concrete or polyester concrete is needed to adjust the profile grade and achieve a continuous cross slope.

5.5.2 TYPICAL SECTIONS AND SPAN LENGTHS

Typical voided slab sections shown in Figure 5.5-1 are based on the AASHTO/PCI Sections, as described in the Precast Concrete Institute (PCI) Bridge Design Manual (PCI, 2014). Four section types with the designations SI through SIV represent depths ranging from 12 to 21 inches, respectively. The section dimensions illustrated in Figure 5.5-1 can be adjusted for non-standard and variable slab widths without a significant increase in cost if the strands are straight and the width is less than 48 inches (PCI, 2014). Since prestressing strands are straight, stresses can be controlled using partial debonding, as discussed in Section 5.3.2.

For preliminary design, the section can be taken from Table 5.5-1. The section should be verified to meet all strength and service limit states including deflection requirements. Note that the service limit deflection analysis is not required if the minimum depths are met, per Article 2.5.2.6.3 (AASHTO, 2017). Voided slabs are subject to the requirements for precast concrete box slabs. The minimum depth (including composite topping slab) is $0.030L$ and $0.025L$ for simple spans and continuous spans, respectively, where L is the span length.

Voided slabs are typically placed so the soffit matches the cross-slope and the average profile grade of the span. Since the sections are wide relative to the depth, the slabs are stable at cross slopes up to ten percent, which is the upper limit specified in the Highway Design Manual Section 202.2 (Caltrans, 2020). The ends of the slabs can be supported with thin neoprene strips on a non-level bent cap and abutment seats. Greased bearings are not recommended.



Dimensions (inches)

Type	*L*	*H*	*L₁*	*L₂*	No. of Voids	*D₁*	*D₂*
SI-36	36	12	–	–	0	–	–
SII-36	36	15	10.5	7.5	2	8	–
SIII-36	36	18	10.5	7.5	2	10	–
SIV-36	36	21	10	8	2	12	–
SI-48	48	12	–	–	0	-	–
SII-48	48	15	10	14	3	8	8
SIII-48	48	18	9.5	14.5	3	10	10
SIV-48	48	21	10	14	3	12	10

Figure 5.5-1 Solid and Voided Slabs (PCI 2014)

Table 5.5-1 Recommended Span Lengths for Preliminary Design

Type	Depth (in.)	Span (ft)**	
		Without Concrete Deck	With Concrete Deck*
SI	12	30	40
SII	15	40	45
SIII	18	45	50
SIV	21	50	55

*Based on a 6-inch cast-in-place concrete deck.
**Simple supports assumed.

During fabrication and subsequent curing operations, the slabs will deflect vertically upward (camber) and horizontally (sweep), as shown in Figure 5.5-2. These deflections will not be uniform for all slabs within a span and must be accounted for in the design.

Sweep is caused by form and prestressing strand misalignment, strand tensioning variability, thermal effects due to sun exposure on one side, and improper storage at the fabrication site. Since differential sweep could result in an imperfect fit, a small gap should be specified between slabs. This gap should be at least 1.5 times the sweep allowed in Section 10.11 of the *Tolerance Manual for Precast Prestressed Concrete Construction, MNL-135-00* (PCI, 2000), which is referenced in *Standard Specifications* Section 90-4.03 (Caltrans, 2018).

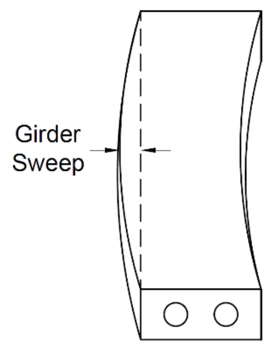


Figure 5.5-2 Schematic Showing Slab Sweep

Camber is a result of eccentrically applied prestress in the slab. Differential camber could result in a non-uniform cross slope and misalignment of the prestress tendons in the transverse diaphragms.

5.5.3 LONGITUDINAL JOINTS

Longitudinal joints between slabs have a keyway that is filled with non-shrink Portland cement grout or Ultra-High-Performance Concrete (UHPC). The grouted keyway creates a water barrier, which can be designed to resist relative deflection of the precast (PC) units under applied loads, as discussed in Section 5.5.5.

For transverse post-tensioned longitudinal joints with non-shrink grout, Article 5.12.2.3.3c requires transverse post-tensioning distributed uniformly along the longitudinal joint to be not less than 0.25 ksi after all losses with a compressed area of the joint not being less than 7.0 inches deep. The intent of this recommendation is to (1) minimize tensile stresses that cause longitudinal cracking at the joints that allow water to penetrate and (2) transfer shear across the joint to prevent differential deflections between slabs and reflective cracks in overlays.

Longitudinal joint details shown in Figure 5.5-3 include a key, with a depth of 7.0 inches, that is filled with non-shrink grout. A foam seal is needed to prevent grout from leaking during placement. Increasing the keyway depths will not significantly improve performance and reduces the prestress acting on the transverse diaphragms. Note that UHPC joints with relatively short reinforcement extensions into the key can transfer shear and prevent relative deflection without transverse post-tensioning, as discussed in Section 5.5.5.

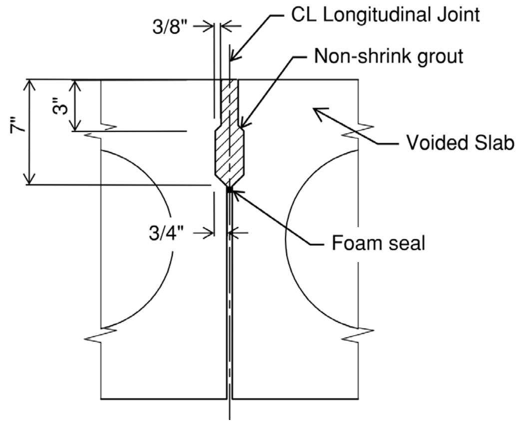


Figure 5.5-3 Recommended Longitudinal Joint Details

5.5.4 OVERLAYS AND CONCRETE DECKS

As discussed, precast slabs are cast on a flat surface. Depending on the span length and the amount of prestressing, the slabs will deflect upward. Further, long-term deflection of precast slabs can be variable resulting in minor elevation differences between slabs over time. Consequently, the desired driving surface profile may not be achievable without an overlay or deck grinding. Overlays can consist of the following:

Asphalt concrete: Asphalt concrete overlays are not recommended for new bridge construction. Although asphalt concrete has a relatively low initial construction cost, it is permeable and can trap moisture on the bridge deck, which could lead to deck deterioration and reduced service life. Further, asphalt concrete overlays prevent inspection of the deck. If asphalt concrete is specified, concrete expansion dams are required at bridge joints to prevent reflective cracking at the ends of the spans.

Polyester Concrete: Although a polyester concrete overlay does not provide structural resistance, it is durable, waterproof, and provides a reliable driving surface within hours of application that make it ideal for Accelerated Bridge Construction (ABC). With a significant tensile capacity and a relatively low elastic modulus, polyester concrete overlays stretch without cracking, thus improving the long-term performance of the bridge deck.

A minimum overlay thickness between one and three inches is recommended in *Bridge Design Memo* (Caltrans). Since polyester concrete requires specialized equipment and personnel to produce and place, the cost can be significant. Further, increasing the thickness of polyester concrete overlays will not necessarily improve the performance. Consequently, the thickness of polyester concrete overlays should be minimized. However, overlays thicker than three inches could be warranted for the following reasons:

- Achieving the desired profile grade requires variation in thickness of the overlay unless a vertical curve is specified that matches the camber of the slab after erection.

- Camber variation between slabs can be significant. Therefore, the overlay thickness should be large enough to accommodate this variability to create a constant cross slope. Alternatively, the deck surface could be ground prior to applying the overlay to reduce the overlay thickness.

Polyester concrete requires a dry concrete surface that has been fully cured to develop a complete bond. Note that the *Standard Specifications* Section 60-3.04B(1)(d) (Caltrans 2018) requires a 28-day concrete cure duration before applying polyester concrete overlay to Portland cement concrete.

Cast-in-Place (CIP) Concrete Deck: A CIP concrete deck reinforced with an orthogonal grid of reinforcement and connected to the slab with stirrups:

- Forms a composite section with the slab, thereby increasing flexure and shear resistance with the interface shear connection provided by the slab stirrups;
- Acts as a water-resistant surface and allows for grading of the profile surface; and
- Provides transverse continuity, which eliminates the need for transverse post-tensioning and intermediate diaphragms.

To achieve transverse continuity, *LRFD Specifications* Article 5.12.2.3.3f (AASHTO 2017) requires that "…the thickness of structural concrete overlay shall not be less than 4.5 in. An isotropic layer of reinforcement shall be provided in accordance with the requirements of Article 5.10.6. The top surface of the precast components shall be roughened." Article 5.10.6 specifies the minimum reinforcement for shrinkage and temperature, and additional reinforcement required to prevent relative deflection should be investigated.

The *PCI Bridge Design Manual* recommends a minimum concrete deck thickness of 5.9 inches. Note that this minimum thickness is typically at midspan and increases toward the supports due to camber, as discussed previously.

For multiple span bridges, the concrete deck can be made continuous for live and superimposed dead loads with the addition of longitudinal reinforcement over the bent cap.

Figure 5.5-4 Voided Slabs with Stirrups Extending into a Cast-in-Place Concrete Deck (Courtesy of Kie-Con)

5.5.5 TRANSVERSE CONTINUITY

Voided slab bridges are constructed by placing slabs side-by-side to create a roadway surface without constructing a deck. Without an adequate transverse connection, these slabs will not deflect equally under live loads. This differential displacement could result in cracking of the grouted longitudinal joint, reflective cracking in the overlay, and the intrusion of water into the joint between the slabs. These cracks should be prevented because water will penetrate between the slabs, which could result in staining and corrosion if subject to the marine environment or deicing chemicals. Further, limiting differential displacement distributes loads to adjacent slabs which improves efficiency.

There are several approaches to limiting this relative displacement and to creating transverse continuity. These approaches include transverse post-tensioned diaphragms, reinforced concrete topping slabs, and grouted longitudinal joints or keyways.

Transverse post-tensioned diaphragms*.* Transverse continuity can be achieved in solid and voided slab spans using transverse post-tensioned diaphragms. These diaphragms, which are located at the ends and locations within the span, reduce relative displacements to within an acceptable amount. Note that post-tensioned diaphragms are not needed if a concrete topping slab is used because the topping slab provides the shear resistance and prevents water penetration, as stated previously.

The *PCI Bridge Design Manual* (PCI, 2014) recommends the diaphragm layout listed in Table 5.5-2 for preliminary design. This publication further states that due to the possibility of cracking at the joints and the resulting loss of stiffness, the use of non-post-tensioned rods is not recommended.

Table 5.5-2 Recommended Diaphragm Locations for Transverse Continuity (PCI, 2014)

Span Length (ft)	No. of Diaphragms	Locations
L ≤ 60	3	Ends and at midspan

Since transverse prestress tendon lengths are relatively short, anchor set losses can be significant. Therefore, post-tensioning rods, per ASTM A722, Type II, are recommended because the anchor set can be as low as 0.0625 inches as compared to 0.375 inches for strand anchorages, per Article C5.9.3.2.1 of the LRFD Specifications (AASHTO 2017).

Post-tensioning ducts are spliced within the grouted blockouts between the slabs, as shown in Figure 5.5-5. Since the blockouts are filled with grout between the slabs prior to post-tensioning, ducts shall be water-tight at the coupler. The Standard Specifications Section 50-1.01D(2)(c) (Caltrans, 2018) requires testing to demonstrate that the ducts can contain an air pressure of 50 psi with minimal leakage prior to grouting. Meeting this criterion can be challenging, as duct splices are to be made within a relatively narrow space, and differential camber between slabs creates minor misalignments of the duct that must be reconciled over a relatively short distance. Therefore, the blockouts should have a minimum width of 3.0 inches measured parallel to the transverse diaphragm to allow adequate room for duct splicing and waterproofing, as shown in Figure 5.5-5.

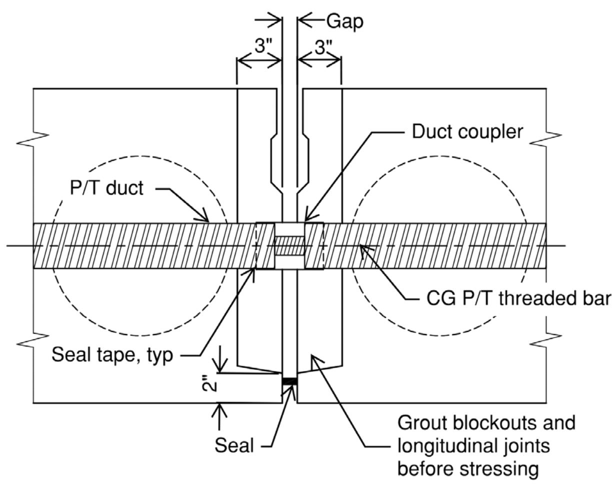


Figure 5.5-5 Blockouts in Transverse Post-Tensioned Diaphragms

Transverse post-tensioning can be designed using the method described in the *PCI Bridge Design Manual* (PCI, 2014). This method provides recommendations for the prestress force required to keep differential deflections within an acceptable limit of 0.02 inches. The required prestress force should compress the full-depth diaphragm sufficiently to remain in compression and uncracked under service loads.

Based on grid analysis models and dimensional parameters that include span length, structure depth, and bridge width, Hanna et al. (2009) developed equations for determining the amount of post tensioning force *P* (kip/ft) per unit length of the bridge.

$$P = \left(\frac{0.9W}{D} - 1.0\right)K_L K_S \leq \left(\frac{0.2W}{D} + 8.0\right)K_L K_S \qquad (5.5.1)$$

Where:

P = Transverse post-tensioning force required (kips/ft)

D = slab depth

W = bridge width

K_L $= 1.0 + 0.003\left(\frac{L}{D} - 30\right)$ correction factors for span-to-depth ratio

K_S = 1.0 + 0.002θ correction factors for skew angle

L = span length

θ = skew angle

The second part of Equation (Eq.) 5.5.1 is intended to control for wide bridges (exceeding 52 feet), where positive bending moments in the diaphragm control the design.

Figure 5.5-6 Transverse Diaphragm Blockout
(Courtesy of Confab, CA.)

UHPC Grouted Keyway Transverse Continuity. Transverse continuity can be achieved through the placement of grout in a formed slot or "keyway" along the longitudinal joint between slabs, as discussed in Section 5.5.3. However, past performance of voided slab bridges has demonstrated that keyways using Portland cement grout are not reliable in resisting relative displacements and providing adequate long-term performance. This poor performance can be attributed to tension and longitudinal cracks that develop between the grout and the slabs.

However, transverse continuity can be achieved with Ultra-High Performance Concrete (UHPC) grout within the keyway and reinforcement extending from the slab into the keyway. Steel fibers in UHPC grout provide confinement resulting in short rebar embedment lengths for dowels extending from the slabs into the keyway. For UHPC connections, an embedment length of eight bar diameters ($8d_b$) is sufficient for most applications including epoxy coated reinforcement.

An example detail using UHPC is shown in Figure 5.5-7, where transverse reinforcement extends beyond the vertical sides that will embed in the keyway. These bars must be installed as bent bars to fit within the vertical side forms and straightened after casting. Alternatively, form-saver mechanical couplings could be used, where the embedded bars are installed after the forms are released.

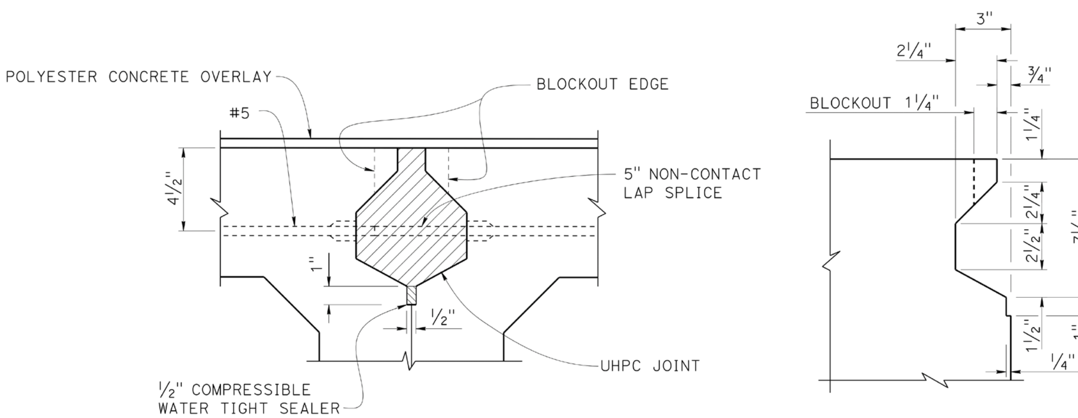
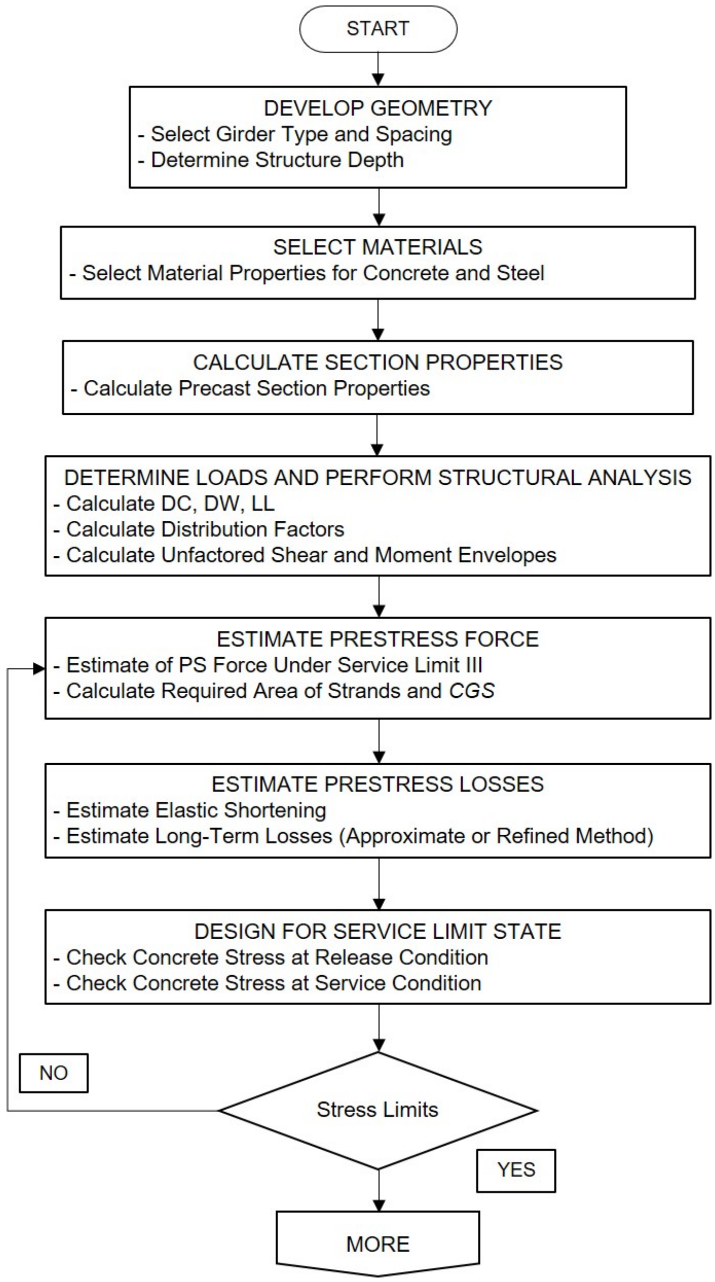


Figure 5.5-7 Longitudinal Keyway Detail with UHPC Connection from the 21st Avenue UC (Replace), Contract No. 03-0h3424

5.5.6 DESIGN FLOW CHART

The following chart shows the typical steps for designing a single-span voided slab bridge or precast box slab bridge connected with transverse post-tensioning.



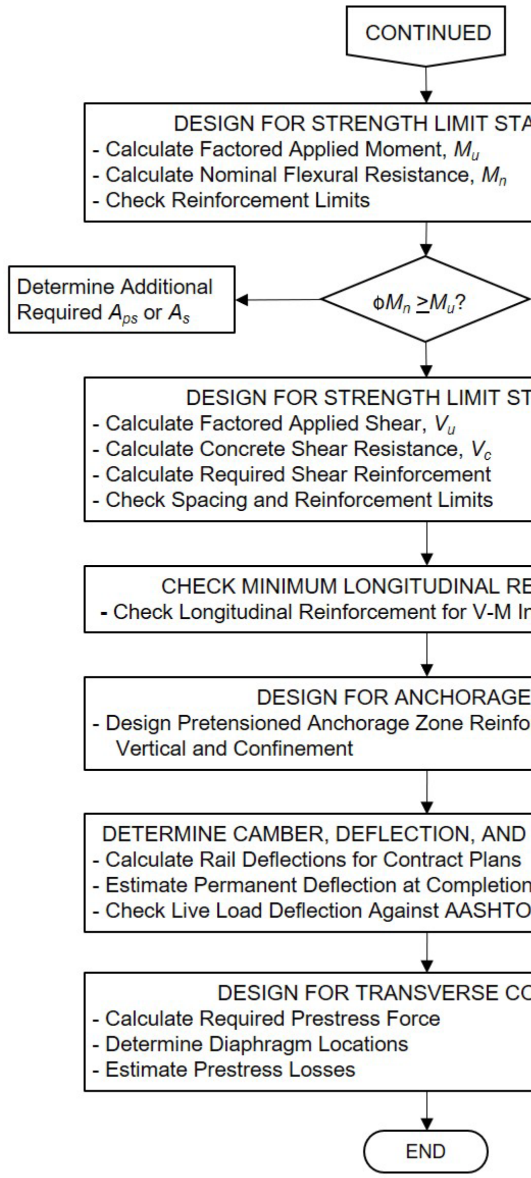


Figure 5.5-8 Precast/Pretensioned Concrete Voided Slab and Box Slab Design Flow Chart with Transverse Post-Tensioning

5.5.7 VOIDED SLAB BRIDGE EXAMPLE

This example illustrates the design procedure for a typical voided slab bridge using AASHTO-CA BDS-8 (AASHTO, 2017; Caltrans, 2019).

To demonstrate the process, a typical 50 ft single-span bridge with no skew is designed using a voided slab with a non-structural polyester concrete overlay. The design live load used for service limit design (Service I and III) is the HL-93 design load, and for strength limit design (Strength II) is the Caltrans P15 permit truck. Elastic flexural stresses for initial and final service limit checks are based on transformed sections. The AASHTO-CA BDS-8 Approximate Method is used to estimate long-term, time-dependent prestress losses based on gross section properties. Shear design is performed using the sectional method.

Major design steps include establishing structural geometry, selecting slab type, selecting materials, performing structural analysis, estimating prestress force, estimating prestress losses, service limit state design, strength limit state design, shear design, anchorage zone design, determining slab deflections, and determining the thickness of the polyester concrete overlay at supports.

5.5.7.1 Problem Statement

A 50-foot-long simple-span bridge is proposed to carry highway traffic across an unlined channel. In order to meet vertical clearance and hydraulic and environmental constraints, a voided slab bridge is recommended. Figures 5.5-9, 5.5-10, and 5.5-11 show the elevation, plan, and typical section views of the bridge, respectively. The effective span length measured from the centerline of bearing to the centerline of bearing is 48 ft and the slab lengths are 50 ft. Slabs will be adjacent to each other and post-tensioned for transverse continuity.

The bridge carries two 12 ft traffic lanes with 8 ft shoulders for a roadway width of 40 ft. The bridge has two 2 ft California ST-75 Bridge Rail barriers and a total deck width of 44 ft. These barriers facilitate ABC, as concrete curbs can be cast prior to erection without significantly increasing the weight or reducing stability during erection. The exterior slabs should have smaller voids to accommodate the barrier dowel bolt assemblies for the bridge rail. The riding surface is comprised of a one-inch-thick polyester overlay that is considered non-structural.

It is required to design a typical interior slab in accordance with AASHTO-CA BDS-8 for all limit states.

Exterior slabs shall be designed for both exterior and interior configurations to allow for future widening and are not illustrated in this example.

5.5.7.2 Select Slab Depth, Type, and Number of Slabs

For a 48 ft span, the AASHTO/PCI voided slab section has been found to be an efficient section. The minimum structure depth-to-span length ratio (D/L) in AASHTO-CA BDS-8 Table 2.5.2.6.3-1 is 0.030 for a simply supported precast box slab, which is applicable to a voided slab, as discussed previously.

Span length, L = 48 ft

$$\text{Assuming: } \frac{\text{Structure Depth, } D_s}{\text{Span Length, } L} = 0.03$$

The minimum depth: $D_s = 0.030(48)12 = 17.3$ in.

Target span ranges for Types SI through SIV, with depths ranging from 12 to 21 inches, are listed in Table 5.5-1. For a span of 48 feet, the SIV section with a depth of 21 inches is a reasonable choice.

Select the 21-inch-deep voided slab (Type SIV) and evaluate the minimum depth-to-span ratio:

$$\frac{D_s}{L} = \frac{1.75}{48} = 0.036 \geq 0.030 \text{ (OK)}$$

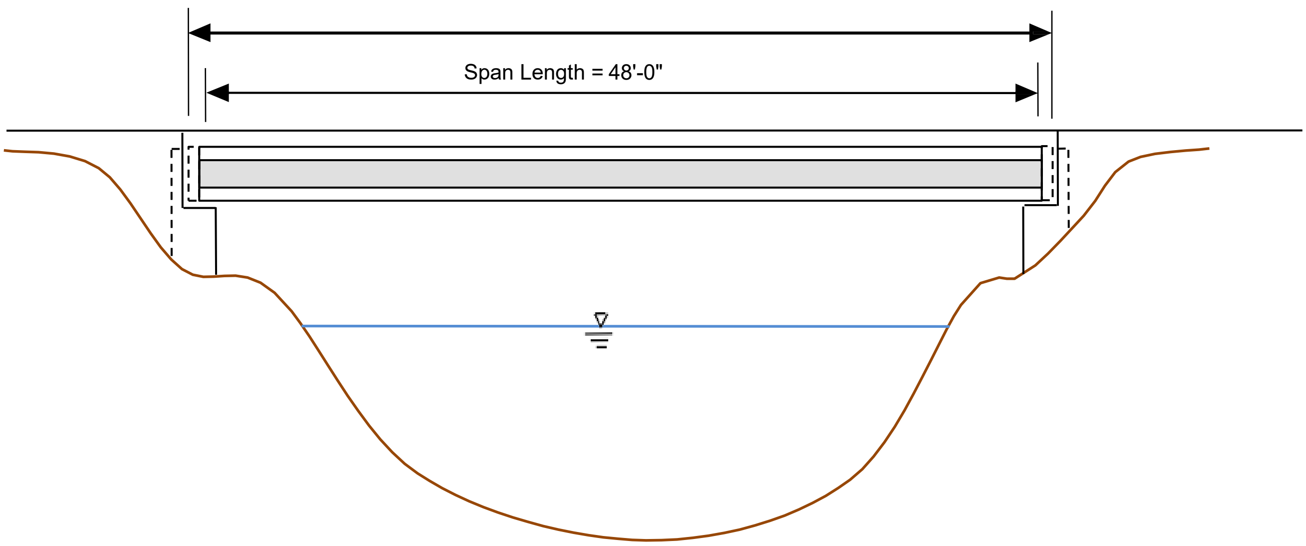


Figure 5.5-9 Elevation View of the Example Bridge

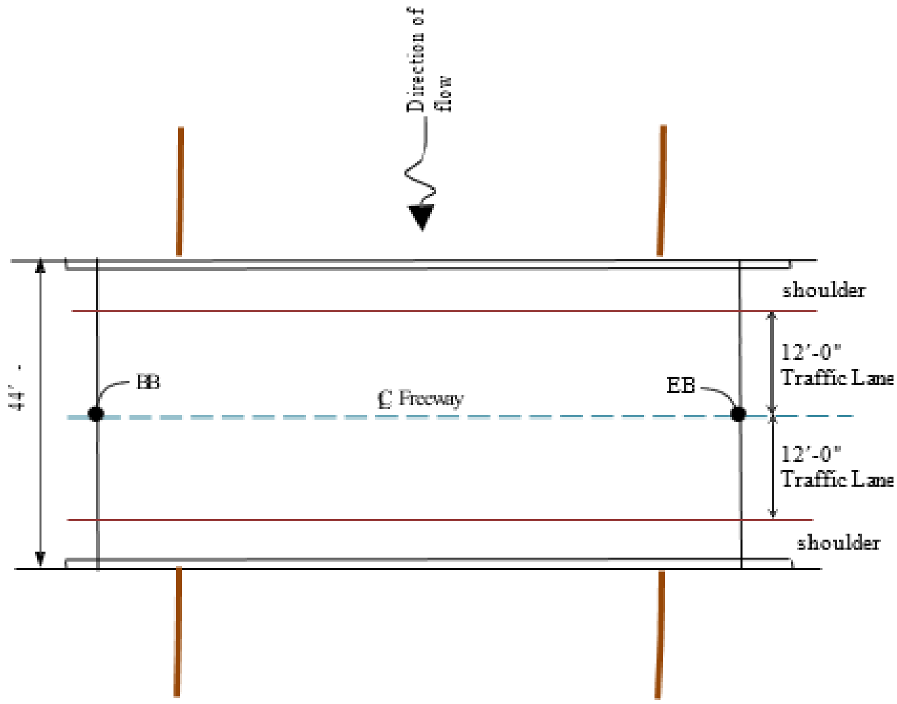


Figure 5.5-10 Plan View of Example Bridge

Although voided slabs have standard widths of 36 and 48 inches, the exterior width dimensions can be adjusted to accommodate custom and variable width slabs if the strands remain straight and the width does not exceed 48 inches. The fewest number of slabs will reduce the erection costs. For bridges with odd widths, one or more 36-inch-wide units can be specified. However, since several slabs are usually cast in the same bed, specifying only one slab with the reduced width is not cost effective.

The slab spacing should be detailed to accommodate differential sweep, which can be caused by strand force variation, void placement tolerance, and sun exposure on one side after casting. A gap is required between slabs to ensure fit on the bearing seats and to ensure the transverse prestressing force is transferred through the diaphragms and not unspecified contact points along the slabs. This gap will include a keyway that is filled with grout. The *Tolerance Manual for Precast and Prestressed Concrete Construction* (PCI, 2000) allows a sweep of 0.375 inches. The gap should be at least 1.5 times the sweep, as discussed in Section 5.5.2.

$$\text{Number of slabs} = \frac{44'\text{ bridge width}}{4'\text{slab width}} = 11$$

$$\text{Gap}\left(G\right) \geq 0.375 \text{ x } 1.5 = 0.563 \text{ in. (use 0.75 in.)}$$

Total width of gaps = 10 gaps x 0.75 in. = 7.5 in. = 0.62 ft

Use 9 – interior 48 in. (4.0 ft) wide slabs and 2 – exterior 44.25 in. (3.69 feet) wide slabs.

Total bridge width, W = 9(4.00) + 2(3.69) + 0.62 = 44.0 feet

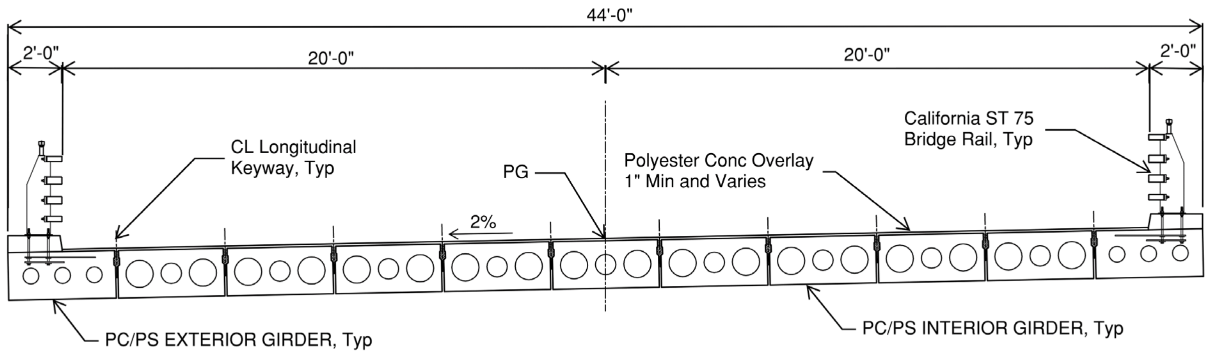


Figure 5.5-11 Typical Bridge Cross Section

5.5.7.3 Establish Loading Sequence

The loading sequence and corresponding stresses for a single-span PC slab are normally considered at three stages, as summarized in Table 5.5-3. All the loading is resisted by the slabs, the overlay is non-structural.

Note: Per Caltrans practice, transportation (shipping and handling) is generally the responsibility of the contractor and PC manufacturer.

Table 5.5-3 Typical Stages of Loading for Single Span PC Slab

Stage	Location	Construction Activity	Loads
I	Casting Yard	Cast and Stressing Slab (Transfer)	DC1 (Slab)
IIA	Bridge Site	Erect Slab	DC1 (Slab), Construction Loads
IIB	Bridge Site	Construct Barrier Rails and Overlay	DC2 (Slab, Barrier Rails), DW (Overlay)
III	Bridge Site	Open to Traffic	DC2 (Slab, Barrier Rails) DW (Overlay) LL (Vehicular Loading, HL-93 or P15)

5.5.7.4 Select Materials

The following materials are selected for the bridge components. The concrete strengths for PC slabs at transfer and at 28 days are assumed based on common practice in California.

- Concrete compressive strength and modulus of elasticity

E_c = Modulus of Elasticity

$= 120{,}000\, K_1 w_c^{2.0} f'^{0.33}_c$ (AASHTO 5.4.2.4-1)

w_c = unit weight of concrete (kcf) = 0.145 kcf for $f'_c \leq 5.0$ ksi(AASHTO Table 3.5.1-1)

$K_1 = 1.0$

At transfer, the required concrete compressive strength (f'_{ci}) can significantly affect the cost, as fabricators rely on the daily use of prestressing beds. Therefore, f'_{ci} should be kept to a minimum, while staying within allowable temporary stresses (PCI, 2014). Caltrans *BDM* 5.3 requires f'_{ci} to be a minimum of 4.0 ksi for pretensioned members.

$f'_{ci} = 4.0$ ksi

$E_{ci} = 120{,}000 \times 1.0 \times 0.145^{2.0} \times 4.0^{0.33} = 3{,}987$ ksi

Concrete compressive strengths (f'_c) of up to 6.0 ksi are readily attainable at 28 days.

$f'_c = 5.0$ ksi

$E_c = 120{,}000 \times 1.0 \times 0.145^{2.0} \times 5.0^{0.33} = 4{,}291$ ksi

- Prestressing steel:

Use 0.6 in. diameter, seven-wire, low-relaxation strands,

A_{ps} = area of each strand = 0.217 in.^{2}

f_{pu} = nominal tensile strength of Grade 270 strand = 270 ksi (AASHTO Table 5.4.4.1-1)

f_{py} = Yield strength = 0.9 f_{pu} = 243 ksi (AASHTO Table 5.4.4.1-1)

f_{pj} = Jacking stress = 0.75 f_{pu} = 202.5 ksi (CA Table 5.9.2.2-1)

E_p = Modulus of elasticity of prestressing steel = 28,500 ksi (AASHTO 5.4.4.2)

- Mild steel – A706 reinforcing steel:

f_y = Nominal yield strength = 60 ksi

E_s = Modulus of elasticity of steel = 29,000 ksi

5.5.7.5 Calculate Gross Section Properties

Gross section properties for a typical interior slab are used to calculate dead loads and deflections, and transformed section properties are used to check flexural stresses at the service limit state. Section properties are based on a slab width of 48 inches and a slab spacing of 48.75 inches to account for the 0.75-inch gap between slabs. Figure 5.5-12 shows the cross section of the slab. The keyways have a negligible effect on section properties and are ignored.

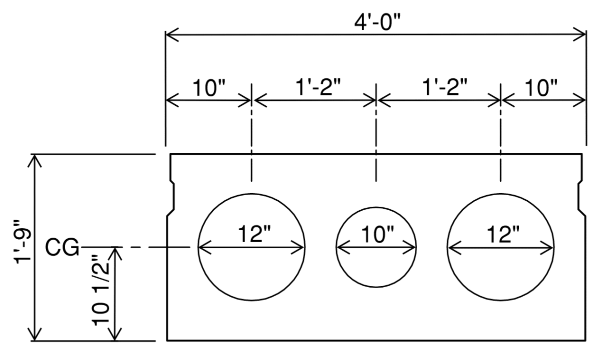


Figure 5.5-12 Voided Slab Cross Section

The area and moment of inertia of the rectangle forming the outer perimeter of the voided slab section are:

$$A_{rect} = 48(21) = 1{,}008 \text{ in.}^2$$

$$I_{rect} = \frac{48(21)^3}{12} = 37{,}044 \text{ in.}^4$$

The area and moment of inertia of voids are:

$$A_{void} = \frac{\pi\left(10^2 + 2(12)^2\right)}{4} = 305 \text{ in.}^2$$

$$I_{void} = \frac{\pi\left(10^4+2\left(12\right)^4\right)}{64} = 2{,}527 \text{ in.}^4$$

The gross cross section properties of the slab are:

$$A_g = A_{rect} - A_{void} = 1{,}008 - 305 = 703 \text{ in.}^2$$

$$I_g = I_{rect} - I_{void} = 37{,}044 - 2{,}527 = 34{,}517 \text{ in.}^4$$

$$S_b \quad \frac{I_g}{H/2} = \frac{34{,}517}{10.5} = 3{,}287 \text{ in.}^3$$

5.5.7.6 Determine Loads

5.5.7.6.1 Permanent Loads

Permanent loads are based on the unit weight of concrete of 0.150 kcf, which includes the weight of prestressing strands, rebar, and forms that remain inside the section.

Voided Slab:

$$w_g = \frac{703}{144}\left(0.150\right) = 0.733 \text{ kip/ft}$$

Distribute dead loads assuming a uniform distribution across the width using the following dead load distribution factor *DFDL*.

DFDL = (Slab width + gap) / (total bridge width)

= 48.75 / (12 x 44) = 0.0923

Type ST-75 barrier rail on both sides of the deck and weigh 250 lbs/ft each:

DC2 = 2(0.250)(0.0923) = 0.046 kip/ft

The wearing surface consists of a one-inch minimum thick polyester concrete overlay. Use a 0.035 kip per square foot (ksf) load that acts over the 40-foot-wide roadway deck distributed using the dead load distribution factor.

DW = 0.035(40)(0.0923) = 0.129 kip/ft

5.5.7.6.2 Live Load

The bridge is designed for HL-93 vehicular live load and the California P15 permit truck.

5.5.7.7 Perform Structural Analysis

5.5.7.7.1 Unfactored Bending Moments and Shears due to *DC* and *DW*

Dead load moments and shears can be obtained from a structural analysis software or can be calculated as follows for simply supported single-span bridges:

Shear at x, $V_x = w(0.5L - x)$

Moment at x, $M_x = 0.5wx(L - x)$

where:

w = uniform dead load, klf

x = distance from the left end of the slab (ft)

L = span length between bearings = 48 ft

Table 5.5-4 Unfactored Bending Moments and Shears due to *DC* and *DW*

Location		Slab Weight (*DC1*)		Barrier Weight (*DC2*)		Wearing Surface (*DW*)	
Dist/Span (X/L)	**Location (ft)**	**Moment (kip-ft)**	**Shear (kip)**	**Moment (kip-ft)**	**Shear (kip)**	**Moment (kip-ft)**	**Shear (kip)**
0 L	0	0.0	17.6	0.0	1.1	0.0	3.1
0.0294 L*	1.41	24.1	16.5	1.5	1.0	4.2	2.9
0.1 L	4.8	76.0	14.1	4.8	0.9	13.4	2.5
0.2 L	9.6	135.0	10.5	8.5	0.7	23.8	1.9
0.3 L	14.4	177.2	7.0	11.2	0.4	31.3	1.2
0.4 L	19.2	202.5	3.5	12.8	0.2	35.7	0.6
0.5 L	24	211.0	0.0	13.3	0.0	37.2	0.0
*Critical Shear Section							

5.5.7.7.2 Unfactored Bending Moments and Shears due to Live Loads

Determine the governing values of shear force and moment envelopes to be distributed to the individual slabs using the AASHTO-CA BDS-8 simplified distribution factor formulas, Article 4.6.2.2.2 and Article 4.6.2.2.3 for moments and shears, respectively. As shown previously, the conditions of Article 4.6.2.2.1 are satisfied for this example bridge. Therefore, the simplified distribution factor formulas are applied to the interior slab design in the following sections.

Live Load Moment Distribution Factor, DFM (for Interior Slabs)

The live load distribution factor for moments (DFM, lanes/slab) for an interior slab is governed by the larger value for one design lane versus two or more design lanes loaded, as shown below.

- One design lane loaded:

$$DFM = k\left(\frac{b}{33.3L}\right)^{0.5}\left(\frac{I}{J}\right)^{0.25}$$ (AASHTO Table 4.6.2.2.2b-1)

Provided the ranges are met:

$35 \le b \le 60$

b = effective slab width = 48.0 in. (OK)

$20 \le L \le 120$

L = span length = 48 ft (OK)

$5 \le N_b \le 20$

N_b = number of slabs = 11 (OK)

where:

$k = 2.5(N_b)^{-0.2} \ge 1.5$

$k = 2.5(11)^{-0.2} = 1.55$ (OK)

$$\left(\frac{I}{J}\right) = 0.54\left(\frac{d}{b}\right) + 0.16$$ (AASHTO Table 4.6.2.2.1-3)

$$\left(\frac{I}{J}\right) = 0.54\left(\frac{21}{48}\right) + 0.16 = 0.40$$

$$DFM = 1.55\left(\frac{48.75}{33.3(48)}\right)^{0.5}(0.40)^{0.25} = 0.214 \text{ lanes/slab}$$

- Two or more design lanes loaded:

$$DFM = k\left(\frac{b}{305}\right)^{0.6}\left(\frac{b}{12.0L}\right)^{0.2}\left(\frac{I}{J}\right)^{0.25} \qquad \text{(AASHTO Table 4.6.2.2.2b-1)}$$

$$DFM = \left(\frac{48.75}{305}\right)^{0.6}\left(\frac{48.75}{12.0(48)}\right)^{0.2}(0.4)^{0.06} = 0.297 \text{ lanes/slab}$$

Therefore, the *DFM* for two or more lanes loaded is larger, and this controls.

Use *DFM* = 0.297 lanes/slab

Live Load Shear Distribution Factor, DFV (for Interior Slabs)

- One design lane loaded:

$$DFV = \left(\frac{b}{130L}\right)^{0.15}\left(\frac{I}{J}\right)^{0.05} \qquad \text{(AASHTO Table 4.6.2.2.3a-1)}$$

Provided the ranges for flexure and additional ranges below are met:

$40{,}000 \leq I \leq 610{,}000$

$25{,}000 \leq J \leq 610{,}000$

I_g = 34,517 in.^{4}

According to Article 4.6.2.2.3a, the distribution factor for shear may be taken as that for moments if the values of *I* or *J* do not comply with the limitations in Table 4.6.2.2.3a-1. Therefore:

DFV = *DFM* = 0.214 lanes per slab for one lane loaded and

DFV = *DFM* = 0.297 lanes per slab for two or more lanes loaded, which controls.

The dynamic load allowance factor (*IM*) is applied to the HL-93 design truck, design tandem, and P15 permit truck only, not to the HL-93 design lane load. Table 3.6.2.1-1 of *California Amendments* (Caltrans, 2019) summarizes the values of IM for various components and load cases.

The live load moments and shears are commonly calculated at tenth points and can be obtained from common structure analysis programs. Spreadsheets can also be used for simple-span structures. In this example, structure analysis software was used to determine the live load moments and shears. The results are tabulated in Table 5.5-5 for HL-93 loading and Table 5.5-6 for P15 loading, respectively. These tables list the envelope values for moments and shears per lane, as well as per slab for the design using distribution factors. The P15 values are derived from single 54-kip axel loads versus 27-kip tandem bogies, as shown in *California Amendments* Figure 3.6.1.8.1.

Table 5.5-5 Unfactored Live Load Moment and Shear Envelope Values due to HL-93 (LL+IM)

Location		Per Lane†		DFM	DFV	Per Slab	
Dist/Span (X/L)	Location (ft)	Moment (kip-ft)	Shear (kip)	(Lane per Slab)	(Lane per Slab)	$M_{(LL+IM)}$ (kip-ft)	$V_{(LL+IM)}$ (kip)
0 L*	0	0	92.5	0.297	0.297	0.0	27.5
0.0294 L**	1.41	126.7	88.8	0.297	0.297	37.7	26.4
0.1 L	4.8	390.7	80.0	0.297	0.297	116.1	23.8
0.2 L	9.6	674.7	67.8	0.297	0.297	200.5	20.1
0.3 L	14.4	852.0	55.9	0.297	0.297	253.2	16.6
0.4 L	19.2	952.4	44.4	0.297	0.297	283.0	13.2
0.5 L	24	961.0	34.3	0.297	0.297	285.6	10.2

*L = Span Length
** Critical section for shear
†These values were obtained from a structural analysis program (Included IM = 33%)

Table 5.5-6 Unfactored Live Load Moment and Shear Envelope Values due to P-15 Truck (LL+IM)

Location		Per Lane†		DFM	DFV	Per Slab	
Dist/Span (X/L)	Location (ft)	Moment (kip-ft)	Shear (kip)	(Lane per Slab)	(Lane per Slab)	$M_{(LL+IM)}$ (kip-ft)	$V_{(LL+IM)}$ (kip)
0 L*	0	0	126.6	0.297	0.297	0.0	37.6
0.0294 L**	1.41	171.2	120.6	0.297	0.297	50.9	35.8
0.1 L	4.8	510.3	106.3	0.297	0.297	151.6	31.6
0.2 L	9.6	826.2	86.1	0.297	0.297	245.5	25.6
0.3 L	14.4	996.3	69.2	0.297	0.297	296.1	20.6
0.4 L	19.2	1117.8	55.7	0.297	0.297	332.2	16.6
0.5 L	24	1215.0	42.2	0.297	0.297	361.1	12.5

*L = Span Length
**Critical section for shear
†These values were obtained from a structural analysis program (Includes IM = 25%)

5.5.7.8 Estimate Prestressing Force and Area of Strands

The minimum jacking force, P_j, and the associated area of prestressing strands can be reasonably estimated based on satisfying the two tensile stress limits at the bottom fiber of the voided slab at the Service III limit state:

- Case A: No tension under permanent loads
- Case B: Tension limited to prevent cracking under total dead and live loads

It should be noted that for Service III, only the HL-93 design live load applies. P15 applies to Strength II but not Service III. The critical location for the bending moment in a voided slab is at the midspan. Gross section properties are used to estimate the effective prestress force.

Calculations for these two critical cases are provided below.

Note: Compression is taken as positive (+) and tension as negative (-).

- Case A: No tension is allowed for components with bonded prestressing tendons or reinforcement, subjected to permanent loads (*DC*, *DW*) only. Set the stress at the bottom fiber equal to zero and solve for the required effective prestress force (at service, i.e., after losses), *P*, to achieve no tension.

$$\frac{P}{A_g}+\frac{Pe_c}{S_b}-\left(\frac{M_{DC1}+M_{DC2}+M_{DW}}{S_b}\right)=0$$

Rearranging the equation:

$$P=\frac{\left(\dfrac{M_{DC1}+M_{DC2}+M_{DW}}{S_b}\right)}{\dfrac{1}{A_g}+\dfrac{e_c}{S_b}}$$

As shown in Table 5.5-4 (DC and DW) and Table 5.5-5 (HL-93 design live load), the maximum moment due to dead load and live load occurs at midspan. Moments on a per slab basis are used for slab design.

where:

P = effective prestress force

M_{DC1} = unfactored moment due to slab self-weight = 211.0 kip-ft

M_{DC2} = unfactored moment due to barrier weight = 13.3 kip-ft

M_{DW} = unfactored moment due to overlay weight = 37.2 kip-ft

S_b = section modulus for the bottom extreme fiber = 3287 in.^{3}

A_g = gross area of slab = 703 in.^{2}

To solve for P, the required effective prestressing force, an estimate of the eccentricity of the slab, e_c, is needed. This can be found by knowing the concrete cover to the stirrups, sizes of stirrups, and the diameters of prestressing strands. For this example, assume non-corrosive exterior exposure and use 1.5 inches of cover for the bottom surface of slab bridges, from Table 5.10.1-1 of *California Amendments* (Caltrans 2019). Assume the stirrups are #4 bars and 0.6-inch-diameter prestressing strands. Use the deformed diameter of reinforcing steel when determining clearances.

Strand offset from bottom of slab $= 1.5 + 0.56 + \frac{0.6}{2} = 2.36$ inches. Round up to the nearest 0.5 inches and use 2.5 inches. Thus, the eccentricity of prestressing force is taken as:

$$e_c = 10.5 - 2.5 = 8.0 \text{ in.}$$

$$P = \frac{\left(\frac{(211+13.3+37.2)(12)}{3287}\right)}{\frac{1}{703}+\frac{8}{3287}} = 247.6 \text{ kip}$$

- Case B: Allowable tension for components subject to the Service III limit state (*DC*, *DW*, HL-93), subjected to not worse than moderate corrosion conditions, and are located in Caltrans Environmental "non-freeze-thaw area" from Table 5.9.2.3.2b-1 of *California Amendments* (Caltrans, 2019):

$= -0.19\lambda\sqrt{f'_c}$; *Note*: λ= 1.0 for normal weight concrete

$$\frac{P}{A_g} + \frac{Pe_c}{S_b} - \frac{(M_{DC1} + M_{DC2} + M_{DW} + 1.0(M_{HL93}))}{S_b} = -0.19\sqrt{f'_c}$$

where:

M_{HL93} = moment due to HL-93 loading at midspan = 285.6 kip-ft (Table 5.5-5)

Rearranging the equation:

$$P = \frac{\left(\dfrac{M_{DC1} + M_{DC2} + M_{DW} + 1.0(M_{HL93})}{S_b}\right) - (0.19)\sqrt{f'_c}}{\dfrac{1}{A_g} + \dfrac{e_c}{S_b}}$$

$$P = \frac{\left(\dfrac{211+13.3+37.2+1.0(285.6)(12)}{3287}\right) - (0.19)\sqrt{5}}{\dfrac{1}{703} + \dfrac{8}{3287}} = 407.8 \text{ kip}$$

The minimum required effective prestressing force, P, at the service level for an interior slab is the larger value from Case A and Case B.

Therefore, $P = P_f = 407.8$ kip/slab.

To determine the minimum required jacking force, an estimate of the prestress losses is needed. Thus, assuming total (immediate and long-term) prestress losses of 20% (of the jacking force), the required jacking force (i.e., just before transfer, ignoring minor losses from jacking to de-tensioning) is:

Minimum jacking force,

$$P_j = \frac{407.8}{0.80} = 509.7 \text{ kip}$$

The required area of prestressing strands, A_{ps}, jacked to 0.75 f_{pu} is:

$$A_{ps} = \frac{509.7}{0.75(270)} = 2.52 \text{in.}^2$$

The number of 0.6-inch diameter strands $\geq \dfrac{2.52}{0.217} = 11.6.$

Rounding to an even number of strands (12) provides symmetry (about a vertical line through the centroid) to produce a uniform stress distribution in the member.

Use twelve 0.6-inch diameter low relaxation Grade 270 strands. The actual area of prestressing strands is:

$$A_{ps} = 12(0.217) = 2.604 \text{ in.}^2$$

The total prestressing force at jacking,

$$P_j = 0.75(270)(2.604) = 527.3 \text{ kip}$$

5.5.7.9 Estimate Prestress Losses

Prestress losses were previously estimated in a very approximate way to determine the number of strands. With an estimated number of strands and layout now determined, prestress losses can be more accurately calculated.

Per AASHTO-CA BDS-8, total losses in prestressing strands are assumed to be the sum of immediate and long-term losses. Immediate losses for strands in PC slab are due to elastic shortening. Long-term losses are primarily due to concrete creep and shrinkage as well as steel relaxation (AASHTO, 2017).

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} \quad \text{(AASHTO 5.9.3.1-1)}$$

where:

Δf_{pES} = change in stress due to elastic shortening loss (ksi)

Δf_{pLT} = losses due to long-term shrinkage and creep of concrete and relaxation of prestressing steel (ksi)

Δf_{pT} = total change in stress due to losses (ksi)

5.5.7.9.1 Elastic Shortening

Immediate elastic shortening losses are easily determined for PC slabs using a closed form solution based on AASHTO-CA BDS-8 Eq. C5.9.3.2.3a-1:

$$\Delta f_{pES} = \frac{A_{ps} f_{pbt} \left(I_g + e_m^{\,2} A_g \right) - e_m M_g A_g}{A_{ps} \left(I_g + e_m^{\,2} A_g \right) + \dfrac{A_g I_g E_{ci}}{E_p}}$$

where:

A_{ps} = area of prestressing steel = 2.604 in.^{2}

A_g = gross area of slab section = 703 in.^{2}

f_{pbt} = stress in prestressing steel immediately prior to transfer

= 0.75(270) = 202.5 ksi, ignoring minor relaxation losses after jacking

E_{ci} = 3,987 ksi

E_p = 28,500 ksi

e_m = prestressing steel eccentricity = 8 in.

I_g = moment of inertia of gross section = 34,517 in.^{4}

M_g = midspan moment due to self-weight of slab

= M_{DC1} = 211.0 k-ft (12 in./ft) = 2,532 k-in.

$$\Delta f_{pES} = \frac{2.604(202.5)\left(34{,}517+8^2(703)\right)-8(2{,}532)(703)}{2.604\left(33{,}746+8^2(703)\right)+\dfrac{703(34{,}517)(3{,}987)}{28{,}500}} = 7.69\,\text{ksi}$$

The initial prestress immediately after transfer = 202.5 – 7.7 = 194.8 ksi

Article C5.9.3.2.3a notes that when transformed section properties are used in calculating concrete stresses, the effects of losses and gains due to the elastic deformation are implicitly accounted for (AASHTO, 2017). Therefore, Δf_{pES} should not be used to reduce the stress in the prestressing strands (and force) for concrete stress calculations at the transfer and the service level.

5.5.7.9.2 Long-Term Losses

AASHTO-CA BDS-8 provides two methods to estimate the time-dependent prestress losses: Approximate Method (Article 5.9.3.3) and Refined Method (Article 5.9.3.4) (AASHTO, 2017). This example compares both methods to estimate long-term, time-dependent prestress losses based on gross section properties.

Approximate Method

Per Article 5.9.3.3, the approximate method is applicable to standard precast, pretensioned members subject to normal loading and environmental conditions, where:

- Members are made from normal-weight concrete (OK)
- Concrete is either steam- or moist-cured (OK)
- Prestressing strands use low relaxation properties (OK)
- Average exposure conditions and temperatures characterize the site (OK)

Because the voided slabs in this example satisfy all the criteria, the Approximate Method can be used. Since Article C5.9.3.3 states that the approximate estimates of time-dependent losses are intended for sections with composite decks only, the approximate method is, therefore, illustrated here for the comparison purpose.

Long-term prestress losses due to creep and shrinkage of concrete and relaxation of steel are estimated using the following formula, in which the three terms correspond to creep, shrinkage, and relaxation, respectively:

$$\Delta f_{pLT} = 10.0 \frac{f_{pi} A_{ps}}{A_g} \gamma_h \gamma_{st} + 12.0 \gamma_h \gamma_{st} + \Delta f_{pR}$$ (AASHTO 5.9.3.3-1)

where:

f_{pi} = prestressing steel stress immediately prior to transfer (ksi)

H = the average annual ambient relative humidity (%)

γ_h = correction factor for relative humidity of ambient air

= $1.7\text{-}0.01H$ (AASHTO 5.9.3.3-2)

γ_{st} = correction factor for specified concrete strength time at of prestress transfer to the concrete member

= $5/(1+f'_{ci})$ (AASHTO 5.9.3.3-3)

Δf_{pR} = an estimation of relaxation loss taken as 2.4 ksi for low-relaxation strand and in accordance with manufacturers recommendation for other types of the strand (ksi)

For this calculation:

f_{pi} = 202.5 ksi

H = Average annual ambient relative humidity = 70%

γ_h = 1.7 – 0.01H = 1.7 - 0.01 (70) = 1

$$\gamma_{st} = \frac{5}{(1+f'_{ci})} = \frac{5}{(1+4)} = 1.0$$

Δf_{pR} = 2.4 ksi for low relaxation strands

$$\Delta f_{pLT} = 10.0 \frac{(202.5)(2.604)}{703}(1)(1) + 12.0(1)(1) + 2.4 == 7.50 + 12.0 + 2.4 = 21.90 \text{ ksi}$$

Total prestress losses

$$\Delta f_{pT} = 7.69 + 21.90 = 29.59 \text{ ksi}$$

$$\frac{\Delta f_{pT}}{f_{pi}} = \frac{29.59}{202.5}(100\%) = 14.6\%$$

The strand stress increases with applied load at service, which is calculated as:

$$\text{Elastic gains} = n\left(\frac{(M_{DC2} + M_{DW} + M_{LL})e_c}{I_c}\right)$$

where:

M_{DC2} = 13.3 kip-ft (Table 5.5-4)

M_{DW} = 37.2 kip-ft (Table 5.5-4)

M_{LL} = 285.6 kip-ft (Table 5.5-5)

$$n = \frac{E_{ps}}{E_c} = \frac{28,500}{3,987} = 7.15$$

e_c = strand eccentricity of slab = 8 inches

$$\text{Elastic gains} = 7.15\left(\frac{12(13.3 + 37.2 + 285.6)8}{34,517}\right) = 6.68 \text{ ksi}$$

f_{pe} = effective stress in prestressing strands using gross non-transformed section properties (service limit state)

$$f_{pe} == 202.5 - 29.6 + 6.68 = 179.6 \text{ ksi}$$

Check prestressing stress limit at service limit state:

$$0.8f_{py} \geq f_{pe} \qquad \text{(Table 5.9.2.2-1, Caltrans 2019)}$$

$$0.8(270)(0.9) = 194.4 \text{ ksi} > 179.6 \text{ ksi} \quad (\text{OK})$$

For gross non-transformed sections, the effective prestressing force after all losses, P_f = 2.604 (179.6) = 467.7 kip

Regarding transformed sections: note that with transformed sections used in subsequent sections of this design example, the prestressing force to be used in concrete stress calculations at transfer is the jacking force, P_j, and the prestressing force to be used in concrete stress calculations at the service level is the final prestressing force, P_f, based on long-term losses only (AASHTO C5.9.3.3).

Effective prestress used with transformed section:

$$f_{pe} = 0.75f_{pu} - \Delta f_{pLT} = 0.75(270) - 21.9 = 180.6 \text{ ksi}$$

$$P_f = f_{pe} A_{ps} = 180.6(2.604) = 470.3 \text{ kip}$$

Refined Method

More accurate values for creep, shrinkage, and relaxation-related losses can be obtained using the provisions of Article 5.9.3.4 and referred to herein as the refined method.

Article 5.9.3.4.4 states the refined method in Articles 5.9.3.4.2 and 5.9.3.4.3 are applicable for non-composite decks, non-composite overlays, and for slabs with no overlays. Values for the time of deck placement may be taken as the time of non-composite deck placement or values the at time of installation of the precast deck without topping. The area of “deck” for these applications is zero.

Assumed durations for calculations:

t_i	= 1 day	Age of concrete at the time of transfer
t_d	= 90 days	Age of concrete at the deck placement
t_f	= 20,000 days	Age of concrete at the final time

The change in prestressing steel stress due to time-dependent loss, Δf_{pLT}, is determined from:

$$\Delta f_{pLT} = \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right)_{id} + \left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS}\right)_{df}$$

(AASHTO 5.9.3.4.1-1)

Prestress loss due to shrinkage of slab concrete between transfer and deck placement is taken as:

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id}$$ (AASHTO 5.9.3.4.2a-1)

where

Concrete shrinkage strain of slab (ε_{bid}) for time between transfer and deck placement (in./in.)

$$\varepsilon_{bid} = k_s k_{hs} k_f k_{td} 0.48x10^{-3}$$ (AASHTO 5.4.2.3.3-1)

The factor for volume to surface ratio of the beam:

$$k_s = 1.45 - 0.13(V/S)$$

$$= 1.45 - 0.13(3.67) = 0.97 \ vs\ 1.0 \ (\text{use } 1.0)$$

V/S is the volume to surface ratio, where 50% of the interior perimeter of the voids are included, per Article 5.4.2.3.2:

$$V/S = \frac{A_g}{2(B+H)+0.5\pi(2D_1+D_1)} = \frac{703}{2(48+21)+0.5\pi(2\times 12+10)} = 3.67 \text{ in.}$$

The humidity factor for shrinkage:

$$k_{hs} = (2.00 - 0.014\,H)$$

$$= (2.00 - 0.014(70)) = 1.020$$

Where H = the average annual relative humidity (assume 70%).

The humidity factor for creep:

$$k_{hc} = 1.56 - 0.008\,H$$

$$= 1.56 - 0.008(70) = 1.00$$

The factor for the effect of concrete strength:

$$k_f = 5/\left(1 + f'_{ci}\right) = 5/\left(1 + 4\right) = 1.0$$

The time development factor:

$$k_{td} = \frac{t}{12\left(\dfrac{100 - 4f'_{ci}}{f'_{ci} + 20}\right) + t} = \frac{89}{12\left(\dfrac{100 - 4 \times 4.0}{4.0 + 20}\right) + 89} = 0.679$$

where t is the maturity of concrete (days) = $t_i - t_d$ = 90 – 1 = 89 days

$$\varepsilon_{bid} = (1.00)(1.02)(1.00)(0.679)\left(0.48x10^{-3}\right) = 0.000333$$

The transformed section coefficient is taken as:

$$k_{id} = \frac{1}{1 + \dfrac{E_p}{E_{ci}} \dfrac{A_{ps}}{A_g}\left(1 + \dfrac{A_g\left(e_{pg}\right)^2}{I_g}\right)\left[1 + 0.7\psi_b\left(t_f, t_i\right)\right]} \quad \text{(AASHTO 5.9.3.4.2a-2)}$$

Where:

e_{pg} = eccentricity of the strand with respect to the centroid of the slab (in.)

$\psi(t_f, t_i)$ = slab creep coefficient at final time due to loading introduced at transfer

For the time between transfer and final:

$$\psi\left(t_f, t_i\right) = 1.9 k_s k_{hc} k_f k_{td} t_i^{-0.118} \quad \text{(AASHTO 5.4.2.3.2-1)}$$

$$k_{td} = \frac{20{,}000}{12\left(\dfrac{100 - 4 \times 4.0}{4.0 + 20}\right) + 20{,}000} = 0.998$$

$$\psi\left(t_f, t_i\right) = 1.9(1.00)(1.00)(1.00)(0.998)(1)^{-0.118} = 1.896$$

$$k_{id} = \frac{1}{1 + \dfrac{28{,}500}{3{,}987} \dfrac{2.604}{703}\left(1 + \dfrac{703(8.0)^2}{34{,}517}\right)\left[1 + 0.7(1.896)\right]} = 0.876$$

$$\Delta f_{pSR} = (0.000333)(28{,}500)(0.876) = 8.30 \text{ ksi}$$

The prestress loss due to creep of the slab concrete between the transfer and the deck placement is taken as:

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \psi_b \left(t_d, t_i\right) K_{id} \qquad \text{(AASHTO 5.9.3.4.2b-1)}$$

Where:

$$f_{cgp} = \frac{P_{pi}}{A_{ti}} + \frac{P_{pi} e_{ti}^2}{I_{ti}} - \frac{M_g e_{ti}}{I_{ti}} = \frac{527}{719} + \frac{527\left(7.8\right)^2}{35{,}519} - \frac{2{,}532\left(7.8\right)}{35{,}519} = 1.084 \text{ ksi}$$

$\psi(t_d,t_i)$ = slab creep coefficient at time of deck placement due to loading introduced at transfer.

$$\psi\left(t_d, t_i\right) = 1.9 k_s k_{hc} k_f k_{td} t_i^{-0.118}$$

$$\psi\left(t_d, t_i\right) = 1.9\left(1.00\right)\left(1.00\right)\left(1.00\right)\left(0.679\right)\left(1\right)^{-0.118} = 1.291$$

$$\Delta f_{pCR} = 1\frac{28{,}500}{3{,}987} 1.084(1.291)\left(0.876\right) = 8.76 \text{ ksi}$$

The prestress loss due to the relaxation of prestressing strands between the transfer and the deck placement.

$$\Delta f_{pR1} = \frac{\mathrm{f_{pt}}}{\mathrm{K_L}}\left(\frac{\mathrm{f_{pt}}}{\mathrm{f_{py}}} - 0.55\right) \qquad \text{(AASHTO 5.9.3.4.2c-1)}$$

Where:

f_{pt} = stress in prestressing strands immediately after transfer, taken not less than $0.55 f_{py}$ (ksi).

K_L = 30 ksi for low-relaxation strands and 7 ksi for other prestress steel unless more reliable manufacturers data is available.

$$\Delta f_{pR1} = \frac{202.5 - 7.7}{30}\left(\frac{202.5\text{-}7.7}{243} - 0.55\right) = 1.63 \text{ ksi}$$

The total time-dependent losses between the transfer and the completion of construction can then be calculated.

$$\left(\Delta f_{pSR} + f_{pCR} + f_{pR1}\right)_{id} = \left(8.30 + 8.76 + 1.63\right) = 18.70 \text{ ksi}$$

The time-dependent losses between the completion of construction and the final is the summation of prestress loss due to shrinkage, creep and relaxation.

Prestress loss due to shrinkage of concrete between the completion of construction and the final.

$$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df} \qquad \text{(AASHTO 5.9.3.4.3a-1)}$$

Concrete shrinkage strain of the slab (ε_{bid}) for the time period between the transfer and the deck placement (in./in.).

$$\varepsilon_{bf} = k_s k_{hs} k_f k_{tdf} 0.48x10^{-3} \qquad \text{(AASHTO 5.4.2.3.3-1)}$$

$$\varepsilon_{bf} = (1.00)(1.02)(1.00)(0.998)\left(0.48x10^{-3}\right) = 0.000489$$

$$\varepsilon_{bdf} = \varepsilon_{bf} - \varepsilon_{bid} = 0.000489 - 0.000333 = 0.000156$$

$$\Delta f_{pSD} = (0.000156)(28{,}500)(0.876) = 3.89 \text{ ksi}$$

Prestress loss due to creep of concrete between the completion of construction and the final.

$$\Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{cgp} \psi_b(t_f, t_i) - \psi_b(t_d, t_i) K_{df} + \frac{E_p}{E_c} \Delta f_{cd} \psi_b(t_f, t_d) K_{df} \qquad \text{(AASHTO 5.9.3.4.3b-1)}$$

Where:

$\psi(t_f, t_d)$ = slab creep coefficient at time of deck placement due to loading introduced at transfer.

$$\psi(t_f, t_d) = 1.9 k_s k_{hc} k_f k_{tdf} t_d^{-0.118}$$

$$k_{td} = \frac{20{,}000 - 90}{12\left(\frac{100 - 4 \times 4.0}{4.0 + 20}\right) + (20{,}000 - 90)} = 0.998$$

$$\psi(t_f, t_d) = 1.9(1.00)(1.00)(1.00)(0.998)(90)^{-0.118} = 1.115$$

The change in the concrete stress at the centroid of prestressing strands due to long-term losses between the transfer and the deck placement, combined with the deck weight and superimposed loads is (ksi):

$$\Delta f_{cd} = -(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})$$

$$\Delta f_{cd} = -(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}) \frac{A_{ps}}{A_g}\left(1 + \frac{A_g e_p^2}{I_g}\right) - \left(\frac{(M_{DC1} + M_{DC2}) e_p}{I_g}\right)$$

$$= -(18.70)\frac{2.604}{703}\left(1 + \frac{703(8.0)^2}{34{,}517}\right) - \left(\frac{12(211 + 13.3)8.0}{34{,}517}\right) = -0.300 \text{ ksi}$$

$$\Delta f_{pCD} = \frac{28{,}500}{3{,}987}1.084\left(1.896\text{-}1.291\right)0.876 + \frac{28{,}500}{4{,}291}\left(\text{-}0.30\right)\left(1.115\right)0.876$$
$$= 2.18 \text{ ksi}$$

The prestress loss due to the relaxation of prestressing strands between the deck placement and the final is taken as:

$$\Delta f_{pR2} = \Delta f_{pR1} = 1.63 \text{ ksi} \qquad \text{(AASHTO 5.9.3.4.3c-1)}$$

The prestress gain due to shrinkage of the deck concrete is taken as zero for this bridge because there is no the composite deck

$$\Delta f_{pSS} = 0 \text{ ksi}$$

The total time-dependent loss is determined as:

$$\Delta f_{pLT} = \left(8.30 + 8.76 + 1.63\right) + \left(3.89 + 2.18 + 1.63 + 0.0\right) = 26.4 \text{ ksi}$$

Total prestress losses are taken as:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} = 7.7 + 26.4 = 34.1 \text{ ksi}$$

$$\frac{\Delta f_{pT}}{f_{pi}} = \frac{34.1}{202.5}\left(100\%\right) = 16.8\%$$

Effective prestress used with gross non-transformed section:

f_{pe} = effective stress in prestressing strands (service level)

= 202.5 – 34.1 = 168.4 ksi

For gross non-transformed sections, the effective prestressing force after all losses,

$$P_f = 2.604 \left(168.4\right) = 438.5 \text{ kip}$$

The effective prestress used with the transformed section:

$$f_{pe} = 0.75\, f_{pu} - \Delta f_{pLT} = 0.75 \left(270\right) - 26.4 = 176.1 \text{ ksi}$$

$$P_f = f_{pe} \left(A_{ps}\right) = 176.1\left(2.604\right) = 458.5 \text{ kip}$$

Stress calculations will use transformed section properties, therefore compare the effective prestress forces from the approximate and refined prestress loss methods.

Approximate method: P_f = 470.3 kip

Refined method: P_f = 458.5 kip

Article C5.9.3.3 states the approximate method is intended only for sections with composite decks. However, these calculations show the difference in methods is only 2.5 percent. Although the results from either method can be used to calculate stresses, this example will use the smaller effective prestress force from the refined method.

5.5.7.10 Design for Service Limit State

Design for the Service Limit State addresses the suitability of the previously estimated strand force and the profile based on Stages I, II, and III. Concrete stresses are checked at the transfer, which may lead to design modifications such as adjusting the strand profile or the initial concrete compressive strength, f'_{ci}. A critical check of stresses at the Service Limit State is normally the check of the tensile stress at the bottom of the slab to prevent possible cracking at Service III (HL-93 design live load).

5.5.7.10.1 Calculate Transformed Section Properties

The use of transformed concrete section properties generally leads to more accurate calculations than use of gross section properties. As this is recognized by Article C5.9.3.2.3a, calculations use transformed section properties.

Transformed section properties are needed for the service limit state design. These include the initial transformed section properties of the slab at the transfer (Stage I) and the final transformed section properties at the erection (Stage IIA) and at the service limit state (Stage III). The section property calculations for these stages are presented below. There is a minor difference in the final and the initial transformed non-composite properties from the use of E_c versus E_{ci}.

The prestressing strand steel area is multiplied by $(n-1)$ to calculate the transformed section properties, where n is the modular ratio between the prestressing strand and concrete.

Table 5.5-7 Transformed Section Properties at Transfer

Section	A_i (in.^{2})	y_i (in.)	$A_i(y_i)$ (in.^{3})	I_i (in.^{4})	$A_i(Y\text{-}y_i)^2$ (in.^{4})
Slab	703	10.5	7,384	34,517	22.8
Strands	16.0	2.5	40	0	980
Total	719	--	7,424	34,517	1,002

$$n-1=\frac{E_{ps}}{E_{ci}}-1=\frac{28{,}500}{3{,}987}-1=6.149$$

6.149 x 2.604 = 16.0

Total A_c =719 in.^{2}

$Y_{Bti} = \Sigma Ay_i \div A_c$ = 7,424 ÷ 719 = 10.32 in.

$Y_{Tti} = d_s - Y_{Bti}$ = 21.00 – 10.32 = 10.68 in.

$I_{ti} = I + \Sigma A(Y\text{-}y_i)^2$
= 34,517 + 1,002 = 35,519 in.^{4}

$S_{Bti} = I_{ti} \div Y_{Bti}$ = 35,519 ÷ 10.32 = 3,441 in.^{3}

$S_{Tti} = I_{ti} \div Y_{Tti}$ = 35,519 ÷ 10.68 = 3,326 in.^{3}

$e_{ti} = Y_{Bti}$ – offset = 10.32 – 2.50 = 7.82 in.

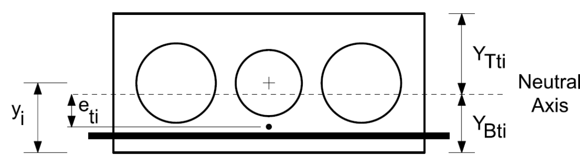


Table 5.5-8 Transformed Section Properties at Final

Section	A_i (in.^{2})	y_i (in.)	$A_i(y_i)$ (in.^{3})	I_i (in.^{4})	$A_i(Y\text{-}y_i)^2$ (in.^{4})
Slab	703	10.5	7,384	34,517	22.8
Strands	14.7	2.5	37	0	899
Total	718	--	7,421	34,517	921

$$n-1=\frac{E_{ps}}{E_{ci}}-1=\frac{28{,}500}{4{,}291}-1=5.642$$

5.642 x 2.604 = 14.7

Total A_c =718 in.^{2}

$Y_{Btf} = \Sigma Ay_i \div A_c$ = 7,421 ÷ 718 = 10.34 in.

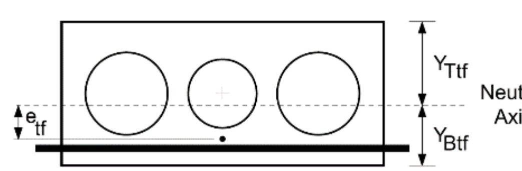
$Y_{Ttf} = d_s - Y_{Btf}$ = 21.00 – 10.34 = 10.66 in.

$I_{tf} = I + \Sigma A(Y\text{-}y_i)^2$
= 34,517 + 921 = 35,438 in.^{4}

$S_{Btf} = I_{tf} / Y_{Btf}$ = 35,438 ÷ 10.34 = 3,429 in.^{3}

$S_{Ttf} = I_{tf} / Y_{Ttf}$ = 35,438 ÷ 10.66 = 3,323 in.^{3}

$e_{tf} = Y_{Btf}$ – offset = 10.34 – 2.50 = 7.84 in.



5.5.7.10.2 Check Concrete Stress at Transfer Condition

The check of concrete stresses at the transfer investigates the suitability of both the prestressing force and the strand profile for the assumed section. Commonly, strands are de-bonded in voided slabs to produce an efficient design that does not overstress the section. In addition, the initial concrete compressive strength, f'_{ci} may be modified.

- Concrete stress limits:
  - Compressive stress limit:

    Stress limit = $0.65f'_{ci} = 0.65\ (4.0) = 2.600$ ksi (AASHTO 5.9.2.3.1a)

  - Tensile stress limit: (AASHTO Table 5.9.2.3.1b-1)
    - In area other than pre-compressed tensile zone without bonded auxiliary reinforcement:

      Stress Limit = $0.0948\sqrt{f'_{ci}} \leq 0.200$ ksi

      Stress Limit = $0.0948\sqrt{4} = 0.190$ ksi

    - In areas with bonded auxiliary reinforcement sufficient to resist the tensile force

      Stress Limit $= 0.24\sqrt{f'_{ci}} = 0.24\sqrt{4} = 0.480$ ksi

Per Article C5.9.3.2.3a, when checking concrete stresses using transformed section properties, the effects of losses and gains due to elastic deformations are implicitly accounted for. Therefore, the elastic shortening loss, Δf_{pES}, should not be subtracted from the strand stress in calculating the prestressing force at transfer (taken as P_j because relaxation losses between jacking and transfer are ignored).

- Check concrete stresses at transfer length section:

  Transfer length = $60(d_b) = 60(0.6) = 36$ in. = 3 ft (AASHTO 5.9.4.3.1)

  d_b = nominal strand diameter (in.)

  P_j = 527 kips

The area, strand eccentricity, and section modulus of the of slab at the transfer with the *CGS* strands = 2.5 in. from the extreme bottom fiber are listed as:

A_{ti} = 719 in.^{2}

e_{ti} = 7.82 in.

S_{Bti} = 3,441 in.^{3}

S_{Tti} = 3,326 in.^{3}

The moment due to the self-weight at a distance from the end of the slab equal to the transfer length of the prestress, based on total slab length, is calculated as:

$$M_{DC1} = 0.5(0.733)(3)(50 - 3) = 51.7 \text{ kip-ft} = 620 \text{ kip-in.}$$

The concrete stress at the top of the slab at the transfer length from the end of the slab is:

$$f_{ci\ top} = \frac{P_j}{A_{ti}} - \frac{P_j e_{ti}}{S_{Tti}} + \frac{M_{DC1}}{S_{Tti}}$$

$$f_{ci\ top} = \frac{527}{719} - \frac{527(7.8)}{3{,}326} + \frac{620}{3{,}326} = \text{-0.321 ksi (Tension)}$$

The tensile stress exceeds the tensile stress for areas without bonded reinforcement but is less than the limit for areas with bonded reinforcement (0.480 ksi). Since the tensile stress at the top of slab exceeds the limit of 0.19 ksi, auxiliary (mild) reinforcement must be provided to the tensile face (top) to resist the total tensile force.

- Determine auxiliary reinforcement:

  Per AASHTO-CA BDS-8 Table 5.9.2.3.1b-1, in areas with bonded reinforcement sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of 0.5 f_y, not to exceed 30.0 ksi. The tensile force is computed as the average tensile stress over the portion of the slab area that is in tension. To find the neutral axis, it is necessary to determine the stress at the extreme tension and compression fibers.

$$f_{ci\ bot} = \frac{P_j}{A_{ti}} + \frac{P_j e_{ti}}{S_{Bti}} - \frac{M_{DC1}}{S_{Bti}}$$

$$f_{ci\ bot} = \frac{527}{719} + \frac{527(7.8)}{3{,}441} - \frac{620}{3{,}441} = 1.75 \text{ ksi} < 2.6 \text{ ksi} \quad \text{(OK)}$$

Locate the axis, *x*, from similar triangles, as shown in Figure 5.5-13.

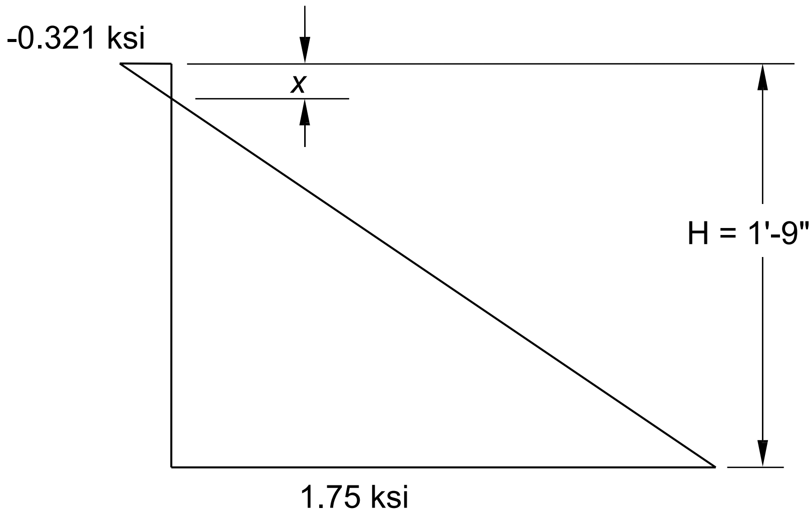


Figure 5.5-13 Concrete Stress Distribution

$$x = h_{slab}\frac{|f_{ci\ top}|}{|f_{ci\ top}| + f_{ci\ bot}} = 21 \times \frac{0.321}{0.321 + 1.75} = 3.25\,\text{in.}$$

$$\text{Depth to void } = \frac{21}{2} - \frac{12}{2} = 4.5 \text{ in.} \quad \text{(OK)}$$

Note: If the void encroached significantly into tensile stress area, a more detailed stress analysis would be required to determine the neutral axis depth and the tensile area of the slab.

Tensile force:

$$T = \frac{f_{ci\ top}}{2} b_{top} x = \frac{0.321}{2}(48)(3.25) = 25.4\,\text{kip} \qquad \text{(AASHTO C5.9.2.3.1b)}$$

Area of mild steel, where $f_s = 0.5f_y \le 30$ ksi:

$$A_s = \frac{T}{f_s} = \frac{25.4}{30} = 0.85 \text{ in.}^2$$

Provide five #4 rebar, with a total area of 1.00 in.^{2}, and debonding of the strands is not required. The bars can run full length in the top of the slab and be located within the hook of the stirrups.

Alternatively, the strand debonding can be used. The maximum number of strands that can be debonded is 33%, per Article 5.9.4.3.3 of the *California Amendments* (Caltrans 2019). An even number of strands should be debonded to facilitate symmetry. Try debonding four strands.

$$\frac{4}{12} \times 100 = 33\% \text{ (OK)}$$

- The check of concrete force in the prestressing strand in the debonded zone is calculated as

  $$P_j = (12 - 4)(0.217)(202.5) = 351\text{kip}$$

  $$f_{top} = \frac{351}{703} - \frac{351(7.8)}{3,326} + \frac{620}{3,326} = \text{-0.152ksi, less than 0.190 ksi}$$

  The limit is in tension (OK)

  $$f_{bot} = \frac{351}{703} + \frac{351(7.8)}{3,441} - \frac{620}{3,441} = 1.11\text{ ksi} < 2.60\text{ ksi (OK)}$$

  Since the tensile stress at the top of the slab does not exceed the upper limit of the tensile stress (0.19 ksi), mild steel is not needed.

- The check of the concrete length of debonding is initially assumed. Also, the top and bottom slab stresses at the end of the debonding influence are calculated and compared to the stress limits for concrete. Debonding length extends to the end of the debonding plus the transfer length of 60 d_b. At this location, the jacking force of all strands are effective and the moments include the self-weight.

  Assuming a debonded length of 5.0 feet,

  $$X = 5.0 + 60d_b = 5.0 + 3.0 = 8.0\text{ ft}$$

  $$M_{DC1} = \frac{0.733 \times 8.0}{2}(50.0 - 8.0) = 123.1\text{ kip-ft=1,477 k-in.}$$

  $$f_{top} = \frac{527}{719} - \frac{527(7.8)}{3,326} + \frac{1,477}{3,326} = \text{-0.063 ksi less than 0.190 ksi}$$

  limit in tension (OK)

  $$f_{bot} = \frac{527}{719} + \frac{527(7.8)}{3,441} - \frac{1,433}{3,441} = 1.50\text{ ksi} < 2.60\text{ ksi}(\text{OK})$$

- Check concrete stresses at midspan:

  The values for P_j, e_{ti}, A_{ti}, S_{Bti}, and S_{Tti} at midspan are the same as at the transfer length section.

  Moment at midspan due to the slab weight, based on the total slab length, is taken as:

  $$M_{DC1} = 0.5(0.733)(25)(50 - 25) = 228.9\text{ kip-ft} = 2,747\text{ kip-in.}$$

  $$f_{top} = \frac{527}{719} - \frac{527(7.8)}{3,326} + \frac{2,747}{3,326} = 0.319\text{ ksi} < 2.60\text{ ksi (OK)}$$

$$f_{bot} = \frac{527}{719} + \frac{527(7.8)}{3,441} - \frac{2,747}{3,441} = 1.133 \text{ ksi} < 2.60 \text{ ksi} \quad \text{(OK)}$$

5.5.7.10.3 Check Concrete Stresses at Service Condition

The check of concrete stresses at the service level investigates the suitability of the section to resist service level loads. Of particular importance is the prevention of flexural cracking of the section at midspan for the Service Level III due to the HL-93 Vehicular Live Load. In addition, per requirements of the *California Amendments*, the section shall not develop any tension under permanent loads (only).

- Determine effective prestressing force, P_f:

  Because transformed section properties are used, the effective prestressing force (P_f) acting on the section is calculated using the force at the transfer, P_j, less long-term losses that were estimated using the Refined Method:

  P_f = 462.0 kip

- Concrete stress limits:
  - Compressive stress limits (AASHTO Table 5.9.2.3.2a-1)
    - Compressive stress limits due to unfactored permanent loads (including slab, barrier, and polyester overlay) and the prestressing force.

      Load combination: PS + Perm

      PC slab: 0.45 f'_c = 0.45(5) = 2.250 ksi

    - Compression stress limit due to the effective prestress, permanent, and transient loads (including all dead and live loads).

      Load combination: Service I = *PS* + *Perm* + (*LL* + IM)$_{HL\text{-}93}$

      PC slab: $0.6\phi_w f'_c$ = 0.6(1.0) (5) = 3.000 ksi

  - Tensile stress limit (Table 5.9.2.3.2b-1, Caltrans 2019)
    - For components with bonded prestressing tendons or reinforcement subjected to permanent loads only.

      Load combination: PS + Perm

      PC slab: 0 ksi (no tension allowed)

- Check concrete stresses at midspan:

  For components with bonded prestressing tendons or reinforcement, evaluate the following limit states:

Service III = PS + Perm + 1.0($LL+IM$)$_{HL\text{-}93}$

Note that the live load factor 1.0 to comply with the requirements of AASHTO-CA BDS-8 Table 3.4.1-4.

PC slab: $0.19\lambda\sqrt{f'_c} = 0.19(1.0)\sqrt{5}$ = 0.424 ksi

Bending moments at the midspan are given in Table 5.5-9.

Table 5.5-9 Unfactored Bending Moments at Midspan (per slab)

Location	*M_{DC1} (kip-ft)	*M_{DC2} (kip-ft)	*M_{DW} (kip-ft)	**$M_{(LL+IM)\ HL93}$ (kip-ft)	***$M_{(LL+IM)\ P15}$ (kip-ft)
0.5L	211.0	13.3	37.2	285.6	351.1
*From Table 5.5-4 ** From Table 5.5-5 *** From Table 5.5-6					

- Check compressive stresses at the midspan:

PC slabs are checked for compressive stresses under the following two load combinations:

- Load combination PS + Perm
  - Stress at top of PC slab:

$$f_{top} = \frac{P_f}{A_{tf}} - \frac{P_f e_{tf}}{S_{Ttf}} + \frac{M_{DC1} + M_{DC2} + M_{DW}}{S_{Ttf}}$$

$$f_{top} = \frac{458.5}{718} - \frac{(458.5)7.8}{3{,}323} + \frac{(211.0 + 13.3 + 37.2)(12)}{3{,}323}$$

$$= 0.502 \text{ ksi (compression) OK}$$

Stress at the bottom of PC slab:

$$f_{bot} = \frac{P_f}{A_{tf}} + \frac{P_f e_{tf}}{S_{Btf}} - \frac{M_{DC1} + M_{DC2} + M_{DW}}{S_{Btf}}$$

$$f_{bot} = \frac{458.5}{718} + \frac{(458.5)7.8}{3{,}429} - \frac{(211.0 + 13.3 + 37.2)(12)}{3{,}429}$$

$$= 0.771 \text{ ksi (compression)} < 2.25 \text{ ksi (OK)}$$

Note that both the top and bottom fibers are in compression. This satisfies the requirement of no tension for components subjected to permanent loads only per *California Amendments* Table 5.9.2.3.2b-1.

- Load combination PS + Perm + (LL+IM)HL-93 (Service I)
  - Stress at the top of PC slab:

$$f_{top} = \frac{P_f}{A_{tf}} - \frac{P_f e_{tf}}{S_{Ttf}} + \frac{M_{DC1} + M_{DC2} + M_{DW} + M_{(LL+IM)HL93}}{S_{Ttf}}$$

$$f_{top} = \frac{458.5}{718} - \frac{(458.5)7.8}{3,323} + \frac{(211.0 + 13.3 + 37.2 + 285.6)(12)}{3,323}$$

$$= 1.53 \text{ ksi } (\text{compression}) < 3.00 \text{ ksi OK}$$

- Tensile stresses at the bottom of the slab at the midspan: (Service III)

  This check to prevent cracking at the midspan is normally a critical check that can govern the prestressing force and thus the area of prestressing strands:

- Load combination PS + Perm + 1.0(LL+IM)HL-93

$$f_{bot} = \frac{P_f}{A_{tf}} + \frac{P_f e_{tf}}{S_{Btf}} - \frac{M_{DC1} + M_{DC2} + M_{DW} + 1.0M_{(LL+IM)HL93}}{S_{Btf}}$$

$$f_{bot} = \frac{458.5}{718} + \frac{(458.5)7.8}{3,429} - \frac{(211.0 + 13.3 + 37.3 + 1.0(285.6))(12)}{3,429}$$

$$= -0.228 \text{ ksi } (\text{tension}) < 0.425 \text{ ksi} \quad \text{Limit in tension (OK)}$$

- Prestressing strand stress at midspan in accordance with Article 5.9.2.2.1 (Service III Limit State):

  f_{pe} = stress limit in prestressing strand after all losses

  = 0.80 f_{py} (AASHTO Table 5.9.2.2-1)

  = 0.80(243) = 194.4 ksi

Total loss due to the elastic shortening at the transfer and the long-term:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} = 7.7 + 26.4 = 34.1 \text{ ksi } (\text{refined estimate})$$

Elastic gain due to superimposed dead loads:

$$\left(\frac{(M_{DC2} + M_{DW})e_{tf}}{I_{tf}}\right)\frac{E_p}{E_c} = \left(\frac{(13.3 + 37.2)(12)7.8}{35,438}\right)\frac{28,500}{4,291} = 0.89 \text{ksi}$$

(Note: strand stress due to slab self-weight is included in Δf_{pES})

Elastic gain due to the superimposed live load:

$$1.0\left(\frac{(M_{LL+IM})e_{tf}}{I_{tf}}\right)\frac{E_p}{E_c} = 1.0\left(\frac{(285.6)(12)7.8}{35,438}\right)\frac{28,500}{4,291} = 5.03\text{ksi}$$

Strand stress at service:

$$\begin{aligned} f_{pe} &= f_{pj} - \Delta f_{pT} + 0.89 + 5.03 \\ &= 202.5 - 34.1 + 0.9 + 5.0 = 174.3\ ksi < 194.4\text{ ksi (OK)} \end{aligned}$$

5.5.7.11 Check Fatigue Stress Limit

Article 5.5.3.1 states fatigue of the reinforcement need not be checked for prestressed components designed to have extreme fiber tensile stress due to Strength III Limit State within the tensile stress limit specified in AASHTO-CA BDS-8 Table 5.9.2.3.2b-1. Since the extreme fiber tensile stress is less than the specified limits, a fatigue stress check of the prestressing strand is not necessary (AASHTO, 2017).

For prestressed components in other than segmentally constructed bridges, the compressive stress due to Fatigue I load combination and one-half the sum of the unfactored effective prestress and permanent loads shall not exceed $0.40f'_c$ after losses.

From *California Amendments* Article 3.6.1.4.1 (Caltrans, 2019) the fatigue truck load shall be one design truck as specified in Article 3.6.1.2.2 with a constant axle spacing of 30.0 ft between 32.0-kip axles, with impact. AASHTO-CA BDS-8 Table 3.4.1-1 specifies a 1.75 load factor on the fatigue truck live load, and Table 3.6.2.1-1 specifies *IM* = 15% for the Fatigue 1 Limit State.

Structural analysis software is used to determine the Fatigue 1 Limit State live load with dynamic allowance (*IM*) midspan moment demand.

The unfactored live load moment with *IM* at midspan, per voided slab

$$M_f = 487.6(0.297) = 144.9\text{ kip-ft}$$

The associated top fiber slab stress

$$M_f = 487.6(0.297) = 144.9\text{ kip-ft}$$

$$f_{tgf} = \frac{1.75\text{M}_\text{f}}{\text{S}_\text{Ttf}} = \frac{1.75(144.9)(12)}{3,323} = 0.916\text{ ksi}$$

Midspan top fiber slab compressive stress due to permanent loads and prestress is

$f_{tg} = f_{top} = 0.502$ ksi, per Section 5.5.7.10.3.

Therefore:

$f_{tg} = f_{top} = 0.502$ ksi

$$f_{tgf} + \frac{f_{tg}}{2} = 0.916 + \frac{0.502}{2} = 1.167 \text{ ksi} \leq 0.40\left(f'_c\right) = 0.40\left(5.00\right) = 2.00 \text{ ksi (OK)}$$

This condition should be satisfied at all locations along the slab.

5.5.7.12 Design for Strength Limit State

5.5.7.12.1 Determine Factored Moments

The factored moments at the Strength limit states, M_u, is based on the unfactored moments previously given in Table 5.5-9.

Table 5.5-10 Unfactored Bending Moments at Midspan (per slab)

Location	$^*M_{DC1}$ (kip-ft)	$^*M_{DC2}$ (kip-ft)	$^*M_{DW}$ (kip-ft)	$^{**}M_{(LL+IM)\ HL93}$ (kip-ft)	$^{***}M_{(LL+IM)P15}$ (kip-ft)
0.5*L*	211	13.3	37.2	285.6	361.1

*From Table 5.5-4
** From Table 5.5-5
*** From Table 5.5-6

M_u is taken as the larger of Strength I and II combinations, per Article 3.4.1 (Caltrans,2019). Strength I uses the AASHTO HL-93 vehicular live load, whereas Strength II uses the California P-15 permit truck.

Determine the controlling factored moment (M_u) at the midspan.

- Strength I:

$$M_u = 1.25\left[M_{DC1} + M_{DC2}\right] + 1.5M_{DW} + 1.75[M_{(LL+IM)HL93}]$$

$$M_u = 1.25\left(211 + 13.3\right) + 1.5\left(37.2\right) + 1.75\left(285.6\right) = 836.0 \text{ kip-ft}$$

- Strength II:

$$M_u = 1.25\left[M_{DC1} + M_{DC2}\right] + 1.5M_{DW} + 1.35[M_{(LL+IM)P15}]$$

$$M_u = 1.25\left(211 + 13.3\right) + 1.5\left(37.2\right) + 1.35\left(361.1\right) = 823.6 \text{ kip-ft}$$

- Strength I governs. M_u = 836.0 kip-ft/slab

5.5.7.12.2 Calculate Factored Flexural Resistance

Based on Article 5.6.3, the nominal flexural resistance of the voided slab section is calculated as follows.

- Determine the average prestressing steel stress at the strength limit state:

  In most applications, the average prestressing steel stress at the strength limit state can be easily determined from AASHTO Eq. 5.6.3.1.1-1, applicable to typical PC slab sections that use bonded tendons and have an effective stress with $f_{pe} \geq 0.5\ f_{pu}$. When more precise calculations are required, the strain compatibility method can be used (AASHTO 5.6.3.2.5).

  For this example, the effective prestress (after all losses),

  $f_{pe} = 176.1\ ksi > 0.5\ f_{pu} = 135$ ksi, thus Eq. 5.6.3.1.1-1 is applicable. The average stress in prestressing strand at the strength limit state

  $$f_{ps} = f_{pu}(1 - k\frac{c}{d_p}) \quad \text{(AASHTO 5.6.3.1.1-1)}$$

  For the low relaxation strand, where $f_{py}/f_{pu} = 0.9$ (Table C5.6.3.1.1-1), the value k is determined as follows:

  $$k = 2(1.04 - \frac{f_{py}}{f_{pu}}) \quad \text{(AASHTO 5.6.3.1.1-2)}$$

  = 0.28 for low relaxation strand,

  Assuming the compressive stress lies above the voids (rectangular stress block develops), the neutral axis depth, c, the distance from the extreme compression fiber to the neutral axis (in.), is determined from the following:

  $$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c \beta_1 b + kA_{ps}\frac{f_{pu}}{d_p}} \quad \text{(AASHTO 5.6.3.1.1-4)}$$

  where:

  A_{ps} = area of prestressing steel = 2.604 in.^{2}

  f_{pu} = specified tensile strength of prestressing steel = 270 ksi

  A_s = area of mild steel tension reinforcement = 0.0 in.^{2}

  A'_s = area of mild steel compression reinforcement = 0.0 in.^{2}

f_s = stress in the mild steel tension reinforcement nominal flexure resistance = 60 ksi

α_1 = stress block factor = 0.85 (AASHTO 5.6.2.2)

f'_s = stress in the mild steel compression reinforcement at nominal flexure resistance

b = effective width of flange in compression = 48 in.

f'_c = compressive strength concrete = 5.0 ksi

d_p = distance from extreme compression fiber to centroid of prestressing tendon = 21 – 2.5 = 18.5 in.

$$\beta_1 = 0.85 - 0.05(f'_c - 4) \geq 0.65$$ (AASHTO 5.6.2.2)

Therefore,

$$\beta_1 = 0.85 - 0.05(5-4) = 0.80 > 0.65 \text{ (OK)}$$

$$c = \frac{2.604(270.0) + 0(60.0)}{0.85(5.0)(0.80)(48.0) + 0.28(2.604)\dfrac{270.0}{18.5}}$$

$$= 4.04 \text{ in.} < 4.5 \text{ in. (rectangular section assumption is OK)}$$

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) = 270\left(1 - 0.28\left(\frac{4.04}{18.5}\right)\right) = 253.5 \text{ksi}$$

Determine factored flexural resistance

M_r = factored flexural resistance = ϕM_n (AASHTO 5.6.3.2.1-1)

where:

ϕ = resistance factor, per AASHTO 5.5.4.2

M_n = nominal flexural resistance for rectangular sections:

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right)$$ (AASHTO 5.6.3.2.2-1)

where:

a = depth of equivalent stress block, in.

a = $\beta_1 c$ = 0.80(4.04) = 3.24 in.

$$M_n = 2.604(253.5)\left(18.5 - \frac{3.24}{2}\right) = 11{,}148 \text{ kip-in.} = 929 \text{ kip-ft}$$

- Determine resistance factor, ϕ:

  According to Article C5.6.2.1 and Figure C5.5.4.2-1 of *California Amendments*

(Caltrans 2019), if the net tensile strain, $\varepsilon_{ti} \geq 0.005$, then the section is defined as tension- controlled and ϕ =1 for flexure.

The net tensile strain is calculated using similar triangles based on the assumed strain distribution through the depth of the section at the strength limit state, as shown in Figure 5.5-14.

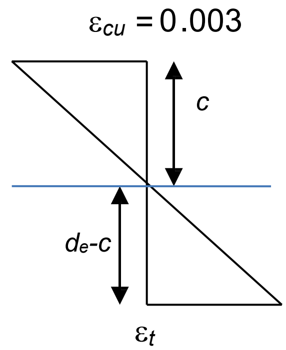


Figure 5.5-14 Assumed Strain Distribution through Section Depth at Strength Limit State

Variables in Figure 5.5-14 are defined as follows:

d_e = effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in.) = d_p

ε_{cu} = failure strain of concrete in compression (in./in.) (AASHTO 5.6.2.1)

ε_t = net tensile strain in extreme tension steel at nominal resistance (in./in.) (AASHTO C5.6.2.1)

By similar triangles:

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_t}{d_e - c}$$

$$\varepsilon_t = \frac{\varepsilon_{cu}}{c}(d_e - c) = \frac{0.003}{4.04}(18.5 - 4.04) = 0.011 \geq 0.005$$

Therefore, the section is tension-controlled, and thus,

ϕ = 1.0. Check flexural capacity of section:

M_r = ϕM_n = 1.0(929) = 929 kip-ft > M_u = 836.0 kip-ft (OK)

5.5.7.13 Check Reinforcement Limits

5.5.7.13.1 Maximum Reinforcement

AASHTO does not have a maximum limit for reinforcing, the current approach involves reducing the flexural resistance factor when the net tensile strain in the extreme reinforcement is less than 0.005. Having a net tensile strain less than 0.004 for precast concrete flexural members is not recommended due to reduced ductility.

5.5.7.13.2 Minimum Flexural Reinforcement

To prevent a brittle failure at the initial flexural cracking, Article 5.6.3.3 requires that all flexural components shall have a sufficient amount of the prestressed and non-prestressed tensile reinforcement to develop a factored flexural resistance, M_r, at least equal to the lesser of: (i) $1.33M_u$ and (ii) M_{cr}.

where:

M_u = controlling factored moment demand

M_{cr} = cracking moment

For this example, the controlling factored moment occurs at the midspan under the Strength I combination.

$$M_u = 836 \text{ kip-ft}$$

$$1.33M_u = 1{,}112 \text{ kip-ft}$$

$$M_{cr} = \gamma_3 \left[\left(\gamma_1 f_r + \gamma_2 f_{cpe} \right) S_c - M_{dnc} \left(\frac{S_c}{S_{nc}} - 1 \right) \right] \quad \text{(AASHTO 5.6.3.3-1)}$$

where:

f_r = modulus of rupture of concrete specified in Article 5.4.2.6

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses), at extreme fiber of section where tensile stress is caused by externally applied load.

M_{dnc} = total unfactored dead load moment acting on the noncomposite section (kip-in.)

S_c = section modulus for the extreme fiber of the composite section where tensile is caused by externally applied loads (in.^{3})

S_{nc} = section modulus for extreme fiber of the non-composite section where tensile is caused by externally applied loads (in.^{3})

Since $S_{nc} = S_c = S_{btf}$,

$$M_{cr} = \gamma_3 \left[\left(\gamma_1 f_r + \gamma_2 f_{cpe} \right) S_{btf} - M_{dnc} \left(\frac{S_{btf}}{S_{btf}} - 1 \right) \right] = \gamma_3 \left[\left(\gamma_1 f_r + \gamma_2 f_{cpe} \right) S_{btf} \right]$$

γ_1 = flexural cracking variability factor 1.6 for other than PC segmental structures

γ_2 = prestress variability factor 1.1 for bonded tendons

γ_3 = ratio of specified minimum yield strength to ultimate tensile strength of reinforcement 1.0 for prestressed concrete structures

$f_r = 0.24\lambda\sqrt{f'_c}$ (AASHTO 5.4.2.6)

where: λ = 1.0 for normal weight concrete (AASHTO 5.4.2.8)

$$f_r = 0.24(1.0)\sqrt{5.0} = 0.537 \text{ ksi}$$

$$f_{cpe} = \frac{P_f}{A_{tf}} + \frac{P_f e_{tf}}{S_{Btf}} = \frac{458.5}{718} + \frac{458.5(7.8)}{3{,}429} = 1.69 \text{ ksi}$$

$$M_{cr} = 1.0\left[\left(1.6(0.537) + 1.1(1.69)\right)(3{,}429)\right]$$
$$= 9{,}305 \text{ kip-in. } 775 \text{ kip-ft}$$

Since $1.33M_u = 1{,}112$ kip-ft $> M_{cr} =$ 775 kip-ft; M_{cr} controls

$M_r =$ 929 kip-ft $\geq M_{cr} = 775$ kip-ft (OK)

5.5.7.14 Design for Shear

The shear design of PC slabs in this example is performed using the sectional design method of Article 5.7.3. However, the General Procedure for Shear Design with Tables is used to determine β and θ, per Appendix B5 of *AASHTO-CA BDS-8* and *California Amendments* 5.7.3.4.2. A design flow chart is provided in Figure CB5.2-5 of Appendix B5 of AASHTO-CA BDS-8.

PC slabs are designed by comparing the factored shear forces (envelope values) and the factored shear resistances at a number of sections along their length, typically at tenth points along the member length and at additional locations near supports. The shear resistance, V_n as specified in Article 5.7.3, consists of the sum of three components:

- Concrete component, V_c, that relies on stress capacity in the concrete
- Steel component, V_s, that relies on the tensile stresses in the transverse reinforcement
- Prestressing component, V_p, the vertical component of the prestressing force for harped strands

This example illustrates shear design only at the critical section.

5.5.7.14.1 Determine Critical Section for Shear Design

For the common situation near supports where the reaction force in the direction of the applied shear introduces a compression into the end region of a member, Article 5.7.3.2 specifies that the location of the critical section for shear shall be taken at a distance, d_v, the effective shear depth, from the internal face of the support.

- Determine effective shear depth, dv, which is the distance between resultants of tensile and compressive forces due to flexure is illustrated in Figure 5.5-16 and defined as follows:

  dv = de - a/2, but not less than the greater of (0.9de, 0.72h) (AASHTO 5.7.2.8)

  The effective depth from the extreme compression fiber to the centroid of the tensile reinforcement

  de = hc - ybts

  where:

  hc = depth of slab

  ybts = centroid of all tensile reinforcement

  The depth of the compression block, a, at the location of dv can be computed using the procedure presented in the flexure design section. It is found that a = 3.24 in.

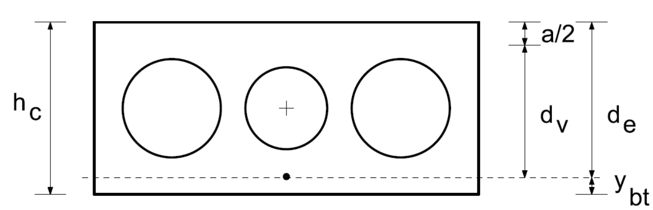


Figure 5.5-15 Definitions of y_{bt}, d_e, and d_v

$$d_e = 21 - 2.5 = 18.5 \text{ in.}$$

$$d_v = 18.5 - \frac{3.24}{2} = 16.88 \text{ in.}$$

$$0.9d_e = 0.9(18.5) = 16.65 \text{ in.}$$

$$0.72h = 0.72(21) = 15.12 \text{ in..}$$

Therefore, use d_v = 16.88 in. (1.41 ft)

5.5.7.14.2 Calculate Factored Shears

Unfactored shears and corresponding factored moments at d_v of 1.41 ft away from the face of the support are summarized in Table 5.5-11.

Table 5.5-11 Unfactored Shears and Bending Moments at d_v

Shear	V_{DC1}	V_{DC2}	V_{DW}	$V_{(LL+IM)HL93}$	$V_{(LL+IM)P15}$
(kip)	16.6	1.0	2.9	26.4	35.8
Associated Moment	M_{DC1}	M_{DC2}	M_{DW}	$M_{(LL+IM)HL93}$	$M_{(LL+IM)P15}$
(kip-ft)	24.0	1.5	4.2	37.6	50.9
	Table 5.5-4			Table 5.5-5	Table 5.5-6

Apply Strength I and Strength II Load Combinations to determine which controls for V_u:

Strength I:

$$\begin{aligned} V_u &= 1.25(V_{DC1}+V_{DC2}) + 1.5V_{DW} + 1.75V_{(LL+IM)HL93} \\ &= 1.25(16.6 + 1.0) + 1.5(2.9) + 1.75(26.4) \\ &= 72.5 \text{ kip} \end{aligned}$$

Strength II:

$$\begin{aligned} V_u &= 1.25(V_{DC1}+V_{DC2}) + 1.5V_{DW} + 1.35V_{(LL+IM)P15} \\ &= 1.25(16.6 + 1.0) + 1.5(2.9) + 1.35(35.8) \\ &= 74.7 \text{ kip (controls)} \end{aligned}$$

Corresponding Strength II factored moment:

$$\begin{aligned} M_u &= 1.25\ (24.0 + 1.5) + 1.5(4.2) + 1.35(50.9) \\ &= 107 \text{ kip-ft} \end{aligned}$$

5.5.7.14.3 Calculate Contribution of Concrete

The concrete contribution to the shear resistance is determined from the following equation:

$$V_c = 0.0316\lambda\beta\sqrt{f'_c}\,b_v d_v \quad \text{(AASHTO 5.7.3.3-3)}$$

where:

β = factor indicating the ability of diagonally cracked concrete to transmit tension and shear

b_v = effective web width taken as the minimum section width between the voids(in.)

d_v = effective shear depth (in.)

For the General Procedure of AASHTO-CA BDS-8 Appendix B5, the value of β is based on the net longitudinal tensile strain, ε_x, at the mid-depth of the section for the normal case in which code-minimum transverse reinforcement is provided.

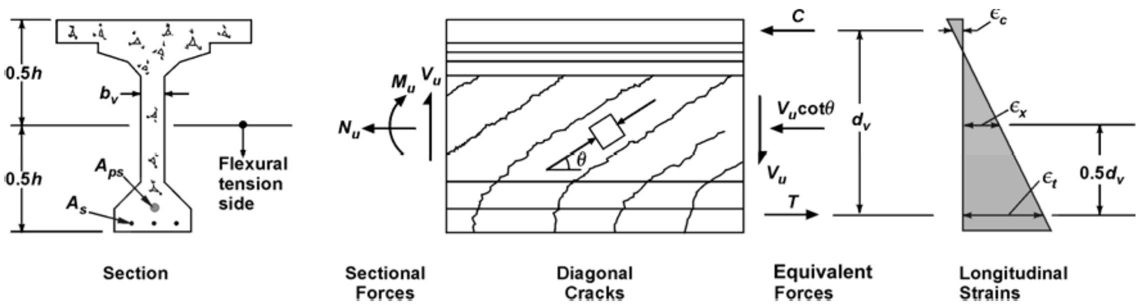


Figure 5.5-16 Shear Parameters for Section Containing at Least Minimum Amount of Transverse Reinforcement, V_p = 0 (AASHTO Figure B5.2-1)

- Determine ε_x:

  For the General Procedure, the longitudinal strain, ε_x, at the mid-depth of the section might be determined using one of the two equations:

  - Eq. B5.2-3 when the strain is tensile (positive)
  - Eq. B5.2-5 when the strain is compressive (negative)

  The value of 0.5 cotθ may be taken equal to 1 (i.e., θ may be taken as 26.6°) initially during iterations for θ and β, and may also be assumed constant to avoid iterations, without significant loss of accuracy.

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right)}{2(E_sA_s + E_pA_{ps})} \qquad \text{(AASHTO B5.2-3)}$$

where:

ε_x = longitudinal strain at the mid-depth of the member

M_u = factored moment at the section, not to be taken less than V_ud_v = 74.7(16.88)/12 = 105 k-ft < M_u = 107 k-ft,

Therefore, use

M_u = 107 kip-ft for the purposes of calculating ε_x.

N_u = factored axial force, taken as positive if tensile and negative if compressive is 0 kips

V_u = factored shear force = 74.7 kip

V_p = component of prestressing force in the direction of the shear force: positive if resisting the applied shear

= 0 kip (horizontal tendons)

θ = angle of inclination of diagonal compressive stresses

= 26.6° initially assumed, based on taking 0.5 (cot θ) = 1

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete. For the usual levels of prestressing, a value of $0.7f_{pu}$ is appropriate for pretensioned members= 0.7(270) = 189 ksi

A_{ps} = area of prestressing strands on flexural tension side at section= 2.604 in.^{2}

A_s = area of non-prestressed steel on flexural tension side of member = 0.0 in.^{2}

$$\varepsilon_x = \frac{\left(\frac{|107(12)|}{16.88} + 0.5(0) + (74.7 - 0) - 2.604(189)\right)}{2(0 + 28{,}500(2.604))} = -2.30 \times 10^{-3}$$

Since ε_x is negative at this location at the mid-depth, the section is in compression. Therefore, ε_x is calculated as follows, which accounts for the presence of concrete in compression.

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right)}{2(E_cA_{ct} + E_sA_s + E_pA_{ps})} \qquad \text{(AASHTO B5.2-5)}$$

where:

A_{ct} = area of concrete on the flexural tension side of the member

= $A_g/2$ = 703/2 = 352 in.^{2}

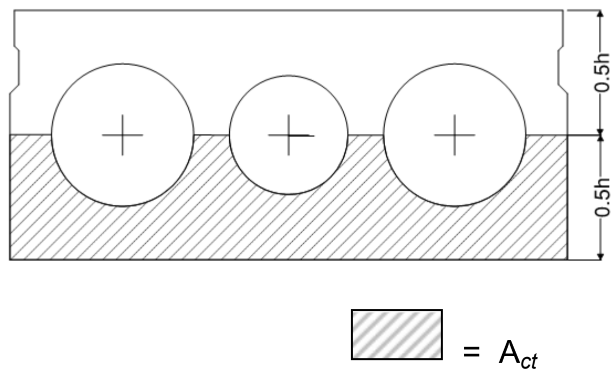


Figure 5.5-17 Definition of A_{ct}

$$\varepsilon_x = \frac{\left(\frac{|107(12)|}{16.88} + 0.5(0) + |74.7 - 0| - 2.604(189)\right)}{2(4,291(352) + 0 + 28,500(2.604))} = -1.08 \; x \; 10^{-4}$$

- Determine β and θ:

For sections with transverse reinforcement equal to or larger than the minimum transverse reinforcement, the value of β (factor for concrete shear contribution) and θ (angle of inclination of diagonal compressive stresses) are estimated through iteration from AASHTO-CS CA BDS-8 Table B5.2-1 as Table 5.5-12. To use this, the ratio (v_u / f'_c) is required in addition to ε_x.

Using ϕ = 0.9 for shear,

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v} \qquad \text{(AASHTO 5.7.2.8-1)}$$

b_v = effective web width (in.)

= beam width – total void width

= 48 – (2x12+10) = 14.0 in.

$$v_u = \frac{|74.7 - 0|}{0.9(14.0)(16.88)} = 0.351 \text{ ksi}$$

$$\frac{v_u}{f'_c} = \frac{0.351}{5.0} = 0.070$$

Table 5.5-12 Values of θ and β for Sections with Transverse Reinforcement (AASHTO Table B5.2-1)

Vu / f`c	**ε_x × 1,000**								
	≤−0.2	**≤−0.1**	**≤−0.05**	**≤0**	**≤0.125**	**≤0.25**	**≤0.5**	**≤0.75**	**≤1**
≤0.075	22.3 6.32	20.4 4.75	***21.0*** ***4.10***	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤0.1	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤0.15	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤0.2	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤0.25	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

From Table 5.5-12 with ε_x = -1.08 x 10^{-4} and v_u / f'_c = 0.070, the values of θ and β could be determined. Although the values to be selected fall between two choices (boxes) in the table, for hand calculations, it is normally simpler and conservative to use the value of θ in the lower row (larger v_u / f'_c) and value of β in column to the right (larger ε_x) of the computed value in the table.

For this design example, the first iteration yields:

$$\theta = 21.0^\circ$$

$$\beta = 4.10$$

The angle θ was initially assumed to be 26.5°, significantly larger than 21.0°. Therefore, another iteration is performed using the angle of 21.0°.

$$\varepsilon_x = \frac{\left(\frac{|107(12)|}{16.88} + 0.5(0) + 0.5|74.7 - 0|\cot(21.0) - 2.604(189)\right)}{2(4,291(352) + 0 + 28,500(2.604))} = -1.08 \ x \ 10^{-4}$$

From Table 5.5-12, the iteration 2 yields the same values for θ and β. Therefore, no further iteration is required, and the following values are used in design at this section:

θ = 21.0°

β = 4.10

- Compute concrete contribution to shear resistance, V_c:

$$V_c = 0.0316\lambda\beta\sqrt{f'_c}b_v d_v \quad \text{(AASHTO 5.7.3.3-3)}$$

$$V_c = 0.0316(1.0)(4.10)\sqrt{5.0}(14.0)(16.88) = 68.5 \text{ kip}$$

5.5.7.14.4 Requirement for Transverse Reinforcement

Check if shear reinforcement is required, i.e.,
when $V_u \geq 0.5\phi\ (V_c + V_p)$ (AASHTO 5.7.2.3-1)

$$V_u = 74.7 \text{ kip} > 0.5(0.9)\ (68.5 + 0) = 30.8 \text{ kip}$$

Therefore, transverse shear reinforcement is required at the critical section.

Required Area of Transverse Reinforcement

The required area of transverse reinforcement is based on satisfying the following design relationship:

$$\frac{V_u}{\phi} \leq V_n = V_c + V_s + V_p \quad \text{(AASHTO 5.7.3.3-1)}$$

Solving this equation for V_s leads to:

$$V_s = \frac{V_u}{\phi} - V_c - V_p = \frac{74.7}{0.9} - 68.5 - 0 = 14.6 \text{kip}$$

The required area of the transverse reinforcement can conveniently be expressed in design as an area per length, i.e., (A_v/s) based on rearrangement of AASHTO Eq. 5.7.3.3-4:

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s}$$

$$\frac{A_v}{s} = \frac{V_s}{f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}$$

where:

s = spacing of transverse reinforcement measured in a direction parallel to the longitudinal reinforcement

A_v = area of shear reinforcement within a distance s

θ = angle of inclination of diagonal compressive stresses

α = angle of inclination of transverse reinforcement to longitudinal axis

= 90° for vertical stirrups

f_y = yield strength of transverse reinforcement

$$\frac{A_v}{s} = \frac{14.6}{60(16.88)\left(\cot 21.0° + \cot 90°\right)\sin 90°} = 0.006\,\frac{\text{in.}^2}{\text{in.}}$$

Using 4-#4 stirrups for the transverse reinforcement,

$$A_v = 0.2\,(4) = 0.80\text{ in.}^2$$

Spacing, s:

$$s = \frac{0.80}{0.006} = 145\text{ in.}$$

Use a spacing of 12 inches (s = 12 inches) near supports. The larger spacing, up to the maximum permitted by AASHTO-CA BDS-8, should be selected beyond the critical section at the discretion of the designer.

This corresponds to a contribution of transverse reinforcement, V_s, to the nominal shear resistance:

$$V_s = \frac{(0.8)(60)(16.88)\left(\cot 21.0°\right)}{12} = 175.9\text{ kip}$$

$$V_n = 68.5 + 175.9 = 244.0\text{ kip} \geq \frac{V_u}{\phi} = 83.1\text{ kip (OK)}$$

Check Maximum Spacing of Transverse Reinforcement

Per Article 5.7.2.6 the spacing of the transverse reinforcement, s, cannot exceed the maximum permissible spacing, s_{max} (i.e., $s \leq s_{max}$). The maximum spacing, s_{max}, depends on the level of the shear stress, v_u. From the previous calculation,

$$v_u = \frac{\left|V_u - \phi V_p\right|}{\phi b_v d_v} = 0.351\text{ ksi}$$

If $v_u < 0.125 f'c$, then $s_{max} = 0.8 d_v \leq 18.0$ in.

$$0.125f'_c = 0.125(5) \ = 0.625\ ksi \ > v_u$$

s_{max} = 0.8d_v = 0.8(16.88) = 13.5 in. (controls)

s_{max} = 18.0 in. (CA 5.7.2.6)

Therefore s_{max} = 13.5 in. Use 12-inch stirrup spacing for the full length of the slab for this example.

Note that a tighter spacing per Eq. 5.7.2.6-2 applies for cases in which:

$$v_u \geq 0.125\ f'_c$$

Check Minimum Transverse Reinforcement

The area of the transverse reinforcement, A_v, shall satisfy

$$A_v \geq 0.0316\lambda\sqrt{f'_c}\,\frac{b_v s}{f_y} \qquad \text{(AASHTO 5.7.2.5-1)}$$

For s = 12.0 in. as provided:

$$A_v = 0.80\text{in.}^2 > 0.0316(1.0)\sqrt{5}\,\frac{14.0(12.0)}{60} = 0.20\ \text{in.}^2 \qquad \text{OK}$$

Therefore, 4-#4 stirrups at 12.0 inches on the center satisfy the minimum transverse reinforcement requirement.

Check Maximum Nominal Shear Resistance

To ensure that the web concrete will not crush prior to the yielding of the transverse reinforcement, Article 5.7.3.3 requires that the nominal shear resistance, V_n, be limited to the smaller of Eq. 5.7.3.3-1 and Eq. 5.7.3.3-2:

$$V_n = V_c + V_s + V_p = 244\ \text{kip} \qquad \text{(AASHTO 5.7.3.3-1)}$$

$$V_n = 0.25f'_c b_v d_v + V_p = 0.25(5)(14)(16.88) + 0 = 295\ \text{kip}$$

(AASHTO 5.7.3.3-2)

Therefore, the nominal shear resistance is 244 kips.

Using the above procedure, the transverse reinforcement along the entire slab can be determined.

5.5.7.15 Check Longitudinal Reinforcement Requirement

The longitudinal reinforcement (including both prestressed and non-prestressed reinforcement on the flexural tension side) at all locations along the slab shall be proportioned to satisfy:

$$A_{ps}f_{ps} + A_s f_y \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta \quad \text{(AASHTO 5.7.3.5-1)}$$

At d_v from the face of the support:

M_u = 107 kip-ft

V_u = 74.7 kip

V_s = 176 kips but not to exceed V_u/ϕ

where:

$$\frac{V_u}{\phi} = \frac{74.7}{0.9} = 83.1\text{kip}$$

V_p = 0 kip

N_u = 0 kip

d_v = 16.88 in.

θ = 21.0°

f_{ps} = f_{pe} = 168.4 ksi (from Section 5.5.7.9.2 for gross section properties)

The determination of the minimum longitudinal reinforcement at d_v from the face of the support is illustrated below.

$$\frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

$$= \frac{|107(12)|}{16.88(1.0)} + 0.5\frac{0.0}{1.0} + \left(\left|\frac{74.7}{0.9} - 0\right| - 0.5(176)\right)\cot 21.0° = 184 \text{ kip}$$

Transfer length, L_t, from the slab end is taken as 60 (strand diameters) = 60(0.6) = 36.0 in. Since the bearing is not yet designed,the face of the support is taken at the center of the support. The center of the support is one foot from the end of the slab. This is more than the distance d_v from the end. Therefore, the strands have not developed the full prestressing force, and the effective prestress is linearly reduced per the following.

$$f_{px} = \frac{f_{pe} l_{px}}{60 d_b} = 168.4\frac{(12.0 + 16.88)}{36.0} = 135.1 \text{ ksi} \quad \text{(AASHTO 5.9.4.3.2-2)}$$

A_{ps} = area of 12 straight strands = 12(0.217) = 2.604 in.^{2}

A_s = 0 in.^{2}

$$A_{ps} f_{ps} + A_s f_y = 2.604(135.1) + 0(60) = 351.8 \text{ kip} > 184 \text{ kip} \quad \text{OK}$$

longitudinal reinforcement requirement is satisfied.

The longitudinal reinforcement requirement shall be satisfied in all locations along the slab. AASHTO Eq. 5.7.3.5-2 shall also be satisfied at the inside edge of the bearing of simple end supports to the section of the critical shear. Article C5.7.3.5 states that forces may be taken at d_v from the face of the support; and the previous calculation satisfies this requirement.

5.5.7.16 Pretensioned Anchorage Zone Reinforcement

The pretensioned anchorage zone reinforcement is designed to resist the splitting and provide the confinement of the concrete around the strand where the strand bonds with the concrete.

- Splitting resistance:

  Article 5.9.4.4 requires the following horizontal reinforcement be provided within the distance h/4 from the end of the slab to provide the splitting resistance to bursting stresses.

$$P_r = f_s A_s \qquad \text{(AASHTO 5.9.4.4.1-1)}$$

where:

f_s = stress in steel not to exceed 20 ksi

A_s = total area of vertical reinforcement located within the distance h/4 from end of slab (in.^{2})

h = overall dimension (width) of the voided slab in the direction in which splitting resistance is being evaluated (in.)

P_r = factored bursting resistance of pretensioned anchorage zone provided by transverse reinforcement (kip), not less than four percent of prestressing force P_i

$$P_r = 0.04\,(P_i) = 0.04(0.75)(270)(2.604) = 21.1 \text{ kip}$$

$$A_s = \frac{21.1}{20} = 1.05 \text{ in.}^2$$

Considering only the two legs of #4 stirrups on the exterior surfaces of the slab:

$$\text{Number of bars required} = \frac{1.05}{0.2(2)} = 2.6$$

Use three #4 stirrups within $h/4$ (48/4 = 12 in.) from the end of the slab at a spacing of 5 inches.

- Confinement reinforcement:

  Article 5.9.4.4.2 requires reinforcement be placed to confine the prestressing steel in the bottom flange, over the distance 1.5d from the end of the slab, using #3 rebar with spacing not to exceed 6 inches and shaped to enclose the strands.

  Use #4 stirrups at 5 in. on the center, to comply with the splitting requirement, over a minimum distance of 1.5d = 1.5(21) = 32 in. (conservative) from the end of the slab to confine the prestressing strands. Use seven spaces at 5 in. = 35 in.

5.5.7.17 Calculate Deflection and Camber

The following two aspects of the deflection and the camber are calculated in this design example:

- Calculate and specify unfactored instantaneous slab deflections due to the overlay and barrier rails for plan sheets, and
- Verify that live load deflections are with the allowable specified in Article 2.5.2.6.2.

The total deflection of the slab is estimated as the sum of the short-term and long-term deflections. Short-term deflections are immediate and are based on an estimate of the modulus of elasticity and the effective moment of inertia. Long-term deflections consist of deflections at the erection and the deflection at the final stage (may be assumed to be approximately 20 years).

5.5.7.17.1 Calculate Instantaneous Deflections

In this section, the instantaneous, unfactored slab camber and deflections due to the prestressing force and the self-weight of the deck, overlay, and barrier are calculated for the contract plans.

- The initial camber due to the prestressing force at the midspan can be estimated. The deflection, Δ_p, is expressed as,

$$\Delta_p = \frac{P_i e_c L^2}{8E_{ci}I}$$

where:

P_i = total prestressing force immediately after transfer (kips)

= $P_j - \Delta F_{pES}$ = (202.5 – 7.69) (2.604) = 507 kip

E_{ci} = modulus of elasticity of concrete at initial (ksi) = 3,987 ksi

I = initial gross (non-transformed) moment of inertia of the slab (in.^{4})

= 34,517 in.^{4}

e_c = eccentricity of prestressing strands at midspan (in.)

= 10.5 – 2.5 = 8 in.

L = overall slab length = 50(12) = 600 in.

$$\Delta_p = \frac{507(8)(600)^2}{8(3,987)(34,517)} = 1.33 \text{ in. (upward)}$$

- The deflection due to the slab self-weight at the midspan:

  The equation for the deflection of a simply supported slab with a distributed load:

$$\Delta_g = \frac{5 w_g L^4}{384 E_{ci} I}$$

where:

w_g = slab self-weight = 0.733 kip/ft

$$\Delta_g = \frac{5(\frac{0.733}{12})(600)^4}{384(3,987)(34,517)} = 0.75 \text{ in. (downward)}$$

- The deflection at the erection due to the slab self-weight at the midspan after the erection:

L = span length between bearings= 48(12) = 576 in.

$$\Delta_g = \frac{5(\frac{0.733}{12})(576)^4}{384(3,987)(34,517)} = 0.64 \text{ in. (downward)}$$

- The instantaneous deflection due to barriers and overlay weights at the midspan.

  Concrete is assumed to have reached full strength and loads are applied to the design span length = 48.0 ft. This deflection is to be included on the contract plans to facilitate voided slab erection.

w_{br} = 0.046 kip/ft

w_{dw} = 0.129 kip/ft

E_c = 4,291 ksi,

$$\Delta_{br+dw} = \frac{5(\frac{0.046+0.129}{12})(576)^4}{384(4,291)(34,517)} = 0.14 \text{ in. (downward)}$$

- The instantaneous net deflection, or camber, at the transfer:

$$\Delta_p + \Delta_g = 1.33 - 0.75 \ = 0.58 \ in. \ (upward)$$

The instantaneous midspan deflections are summarized in Table 5.5-13, which include multipliers to account for creep between the transfer and the erection from PCI Bridge Design Manual Table 8.7.1-1. The respective multipliers at the erection and at the completion of construction are assumed equal due to the short duration and the minimal creep deflection anticipated between these two milestones. A multiplier of 1.0 is used for the barrier and the overlay for the same reason. The multipliers of PCI Bridge Design Manual Table 7.1-1 are not recommended for estimating long-term deflections, as discussed in PCI Bridge Design Manual Section 8.7.1.

A sample calculation of the deflections at the completion of construction is shown for the prestressing as follows:

$\Delta_{p(completion)} = \Delta_{p(elastic)}$ x multiplier = 1.33 (1.80) = 2.39 in.

Table 5.5-13 Tabulated Midspan Deflections (up is positive)

Item	Release	Multiplier*	Erection	Multiplier*	Completion of Construction
Prestress	1.33	1.80	2.39	1.80	2.39
Slab Self Weight	-0.64	1.85	-1.18	1.85	-1.18
Barrier and overlay	-0.14	--	--	1.00	-0.14
Total			1.21		1.07
*Multiplier, per PCI Bridge Design Manual (2014)					

As shown, the multipliers indicate that the slab will have a net camber at the bottom of the slab of 1.21 inches upward at the erection. After the placement of the barriers and the overlay, the driving surface should match the desired profile grade of the roadway. In the final condition, a camber of 1.07 inches is anticipated along the soffit of the slab. If the profile grade is flat, the overlay will be 1.07 inches thicker at the ends of the slab than at the midspan.

5.5.7.17.2 Compare Live Load Deflection to AASHTO Limit

The slab live load deflection check is estimated using composite section properties and concrete strength at the service and compared to the deflection limit per Article 2.5.2.6.2. It should be noted that the deflection criteria Article 2.5.2.6.2 is optional for California bridges. However, for specific situations, such as bridge widening where the deflection may impair the minimum vertical clearance, the deflection must be accounted for in the design.

Caltrans keeps silence about the optional live load deflection criteria specified in Article 2.5.2.6.2, the deflection loading is specified in Article 3.6.1.3.2 and is the greater of:

- That resulting from the design truck plus impact, ΔLT, or
- That resulting from 25% of the design truck plus impact, ΔLT, taken together with the design lane load, Δ_{LL}

The instantaneous truck live load deflection for a simple span bridge occurs at the midspan due to the HL-93 truck axles placed in the location shown below in Figure 5.5-18:

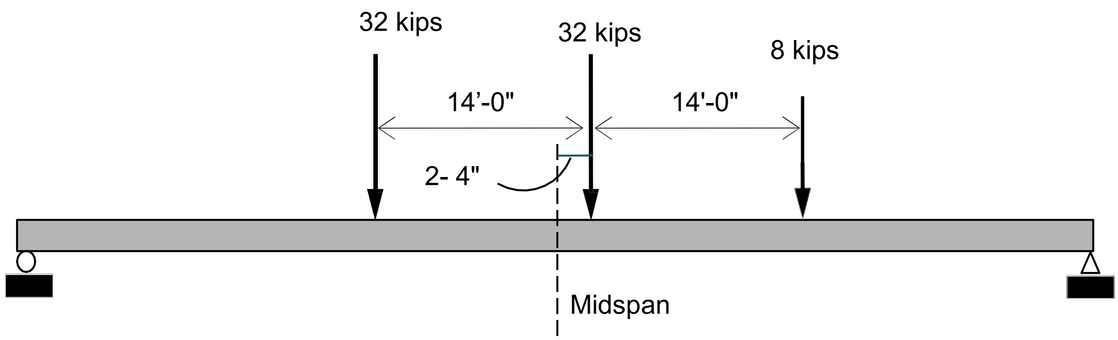


Figure 5.5-18 Position of Truck to Produce Maximum Moment

The truck live load deflection for each lane can be obtained from any structural analysis software program, such as CT Bridge. The deflection for each slab is calculated by multiplying the deflection per lane by the ratio of (number of lanes/numbers of slabs). This ratio can also be estimated using the moment distribution factor (DFM) shown in Sec. 5.5.7.7.2.1, which is the more conservative method.

As shown in Sec. 5.5.7.7.2.1, the DFM for this bridge is 0.297 lanes/slab.

$$\Delta_{LT} = DFM\ (\Delta_{LL\ per\ lane})\ (IM) = 0.297\ (\Delta_{LT\ per\ lane})\ (\text{IM})$$

From structural analysis software, $\Delta_{LL\ per\ lane}$ = 1.70 in. (downward)

$$\Delta_{LT} = 0.297\ (1.70)\ (1.33) = 0.67 \text{ in. (downward)}$$

The lane live load deflection is calculated using the formula for deflection of a simply supported slab with a uniform distributed load.

Design lane load, $w = 0.64DFM = 0.64(0.297) = 0.190$ kip/ft/slab

$$\Delta_{LL} = \frac{5(\frac{0.190}{12})(576)^4}{384(4,291)(35,438)} = 0.15 \text{ in. (downward)}$$

Therefore, the live load deflection is the greater of:

$$\Delta_{LT} = 0.67 \text{ in. (Controls)}$$

$$0.25\Delta_{LT} + \Delta_{LL} = 0.25(0.67) + 0.15 = 0.32 \text{ in.}$$

This instantaneous live load deflection is compared to recommended limitation of *L*/800 in Article 2.5.2.6.2 for general vehicular loading.

$$\frac{L}{800} = \frac{48(12)}{800} = 0.72 \text{ in.} > 0.67 \text{ in.}$$

The live load deflection is less than the AASHTO limit and is acceptable.

5.5.7.18 Check Transverse Post-Tensioning

Article C4.6.2.2.1 states that for bridge type (g), the structure acts as a monolithic unit if sufficiently interconnected. To satisfy this requirement Article 5.12.2.3.3c requires that a minimum average transverse prestress of 0.25 ksi be used. However, the definition of the contact area for the post-tensioning is unclear as to whether it is the shear key, the diaphragm, or the entire slab side surface. Instead of an empirical minimum, El-Remaily (1996) recommends that the entire deck surface be modeled as a rigid assembly of gridwork with adequate post-tensioning to provide a continuous transverse member at the diaphragm locations.

Hanna et al. (2009) developed equations to determine the amount of prestress (*P*) per foot of bridge length of adjacent slab bridge spans. The equations were developed by fitting data points from several grid analyses of various span lengths, structure depths, and bridge widths.

$$P = \left(\frac{0.9W}{D} - 1.0\right)K_L K_S \le \left(\frac{0.2W}{D} + 8.0\right)K_L K_S$$

where:

D = superstructure depth (ft)

W = bridge width (ft)

K_L, K_S = correction factors for span-to-depth ratios and skew, respectively

$$K_L = 1.0 + 0.003\left(\frac{L}{D} - 30\right) = 1.0 + 0.003\left(\frac{48}{1.75} - 30\right) = 0.992$$

$$K_S = 1.0 + 0.002\theta = 1.0 + 0.002(0) = 1.0$$

L = span length

θ = skew angle

$$P = \left(\frac{0.9(44)}{1.75} - 1.0 \right)(0.992)(1.0) = 21.46 > \left(\frac{0.2(44)}{1.75} + 8.0 \right)(0.992)(1.0) = 12.93 \text{ kip/ft}$$

Per Table 5.5-2 the minimum number of diaphragms recommended for bridges with spans less than or equal to 60 feet is three; one at the midspan and one at each end. Since this bridge span is closer to 60 feet and has a shallow structure depth, use four diaphragms: one at each end and at 1/3 points within the span. The shallow depth allows for one prestress rod at mid-depth per diaphragm and using two diaphragms within the span distributes the force and reduces the size of the rods per diaphragm.

Per Hanna et al. (2009) assume that the effective prestressing force after all losses are 55% of the ultimate strength of the rod, f_{pu}.

The maximum spacing of diaphragms, $S = \frac{48}{3}$ = 16.0 ft

Use diaphragms as shown in Figure 5.5-19,

The required force per diaphragm, P_{dia} = 12.93(16.0) = 206.8 kip

150 ksi ASTM A722 PS bar, f_{pu} = 150 ksi

PS bar effective stress, f_{eff} = 0.55(150) = 82.5 ksi

The minimum area of PS bar, $A_{ps} = \frac{P_{dia}}{f_{eff}} = \frac{206.8}{82.5} = 2.51 \text{ in.}^2$

Try 1 3/4" threaded rod PT bar, A_{ps} = 2.60 in.^{2} (from manufacturer data)

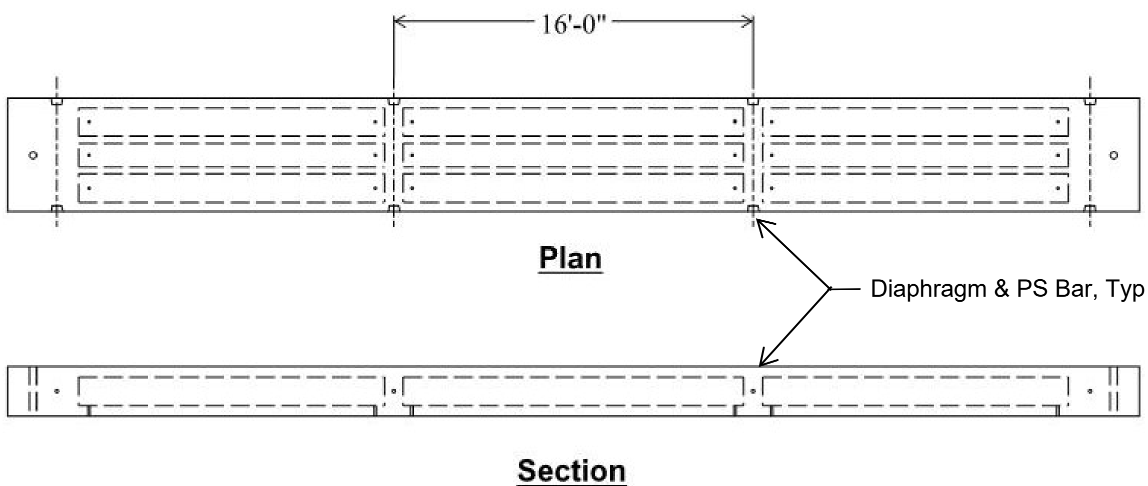


Figure 5.5-19 Slab Plan and Section

5.5.7.18.1 Calculate Transverse Post-Tensioning Losses

Calculating the losses in the transverse post-tensioning is to verify the adequacy of the design and the assumption on the total loss.

$$\Delta f_{pT} = \Delta f_{pF} + \Delta f_{pA} + \Delta f_{pES} + \Delta f_{pLT} \quad \text{(AASHTO 5.9.3.1-2)}$$

where:

Δf_{pF} = loss due to friction (ksi)

Δf_{pA} = loss due to anchorage set (ksi)

Δf_{pES} = change in stress due to elastic shortening loss (ksi)

Δf_{pLT} = losses due to long-term shrinkage and creep of concrete and relaxation of prestressing steel (ksi)

Δf_{pT} = total losses (ksi)

California Amendments Table 5.9.2.2-1 (Caltrans 2019) specifies the maximum jacking stress is 75% of the bar tensile strength (f_{pu}) but is limited by stresses immediately after the seating. Therefore, try a jacking stress of 73% f_{pu} so as not to exceed stresses immediately after the seating:

$$f_{pj} = f_{pbt} = 0.73\ f_{pu} = 0.73(150) = 109.5 \text{ ksi}$$

Estimate the bar length between anchorages for loss calculations:

Anchorage blockout depth = 8 in.

Bar length, x = bridge width – blockouts = 44(12) – 2(8) = 512 in. = 42.67 ft.

Friction loss:

Since the prestress bar is straight and there are no deviations, the friction loss only needs to be calculated at the opposite anchorage on the other side of the bridge.

$$\Delta f_{pF} = f_{pj}(1 - e^{-(\kappa x + \mu\alpha)}) \quad \text{(AASHTO 5.9.3.2.2b-1)}$$

where:

κ = wobble friction coefficient =0.0002/ft (CA Amendments Table 5.9.3.2.2b-1)

μ = Friction factor = 0.30 (CA Amendments Table 5.9.3.2.2b-1)

α = tendon angle change = 0

$$\Delta f_{pF} = 109.5\left(1 - e^{-((0.0002)(42.667)+(0.30)(0))}\right) = 0.93\text{ksi}$$

Anchor set loss:

The anchor set loss can be calculated using the equations:

$$\Delta f_{pA} = \frac{2(\Delta f_{pF})(x_{pA})}{L}$$

$$x_{pA} = \sqrt{\frac{E_p\left(\Delta_{Aset}\right)L}{12\Delta f_{pF}}}$$

where:

x_{pA}	=	Influence length of anchor set (ft)
Δ_{Aset}	=	anchor set length = 0.125 in. (assume 1/8”)
E_p	=	modulus of elasticity of prestressing = 29,000 ksi
L	=	distance to a point of known stress loss = 42.67 ft
Δf_{pF}	=	friction loss at the point of known stress loss = 0.93 ksi

$$x_{pA} = \sqrt{\frac{29{,}000(0.125)42.67}{12(0.93)}} = 117.7\text{ ft}$$

(Note: theoretical influence length extends beyond width of bridge)

The anchor set length can be calculated by using the average anchor set strain multiplied by the length of the tendon. Solving for the anchor set loss results in the following:

$$\Delta f_{pA} = \frac{2(0.93)(117)}{42.67} = 5.13\text{ ksi}$$

***Elastic shortening loss*:**

$$\Delta f_{pES} = 0.5\frac{A_{ps}f_{pbt}\left(I_g + e_m^2 A_g\right) - e_m M_g A_g}{A_{ps}\left(I_g + e_m^2 A_g\right) + \frac{A_g I_g E_{ci}}{E_p}} \qquad \text{(CA C5.9.3.2.3b-1)}$$

where:

Area of one post-tensioning bar, A_{ps} = 2.60 in.^{2}

Post-tension diaphragm width, w = 8.0 in.

Post-tension diaphragm height, h = 21.0 in.

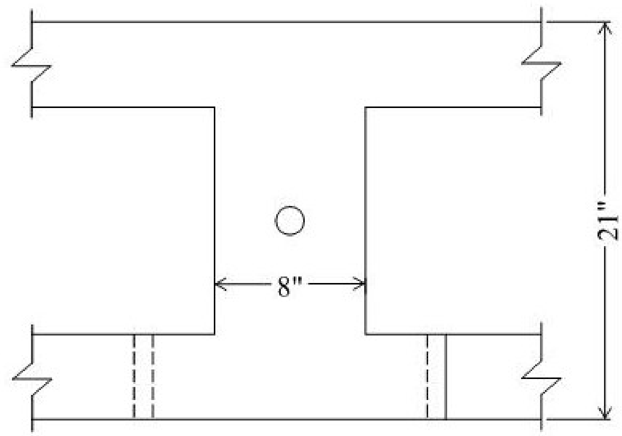


Figure 5.5-20 Slab Transverse Diaphragm

Area of one diaphragm, A_g

$$A_g = w(h) = 8(21) = 168 \text{ in.}^2$$

Moment of inertia of one diaphragm, I_g

$$I_g = \frac{wh^3}{12} = \frac{8(21^3)}{12} = 6{,}174 \text{ in.}^4$$

Concrete modulus of elasticity, E_c = 4,291 ksi (concrete at full strength at time of transverse stressing)

Midspan self-weight moment, M_g = 0 kip-in. (no transverse bending due to self-weight)

Post-tensioning eccentricity, e_m = 0 in. (bar is concentric with diaphragm)

$$\Delta f_{pES} = 0.5\frac{(2.60)(109.5)\left(6{,}174 + (0)(168)\right) - (0)(0)(168)}{(2.60)\left((6{,}174) + (0)(168)\right) + \dfrac{(168)(6174)(4{,}291)}{(29{,}000)}} = 5.18 \text{ ksi}$$

Long term loss:

Δf_{pLT} = 20 ksi (CA 5.9.3.3)

Total post-tensioning loss:

Δf_{pT} = 0.93 + 5.13 + 5.18 + 20.0 = 31.25 ksi

Final transverse post-tensioning force per diaphragm:

The effective post-tensioning stress after all losses,

f_{eff} = 109.5-31.25 = 78.25 ksi

The final diaphragm force, $P_{dia} = A_{ps}(f_{eff})$ = 2.60(78.25) = 203.5 kips

The required diaphragm force, P_{dia} = 206.8 kips, say OK within two percent.

5.5.7.18.2 Check Transverse Post-Tensioning Stresses

Stresses in Post-Tensioning Rods

California Amendments Table 5.9.2.2-1 (Caltrans, 2019) specifies that the maximum stress immediately after seating is 70% of the bar tensile strength (f_{pu}):

f_{ps} = 0.70 f_{pu} = 0.70(150) = 105.0 ksi

Stress at seating = $f_{pbt} - \Delta f_{pA}$ = 109.5 – 5.13 = 104.4 ksi (OK)

California Amendments Table 5.9.2.2-1 (Caltrans ,2019) specifies the maximum stress in the bar after all loses at service is 80% of the bar tensile yield stress (f_{py}):

f_{py} = 0.80 f_{pu} = 0.80(150) = 120 ksi

f_{pe} = 0.80 f_{py} = 0.80(120) = 96.0 ksi (service stress limit after losses)

Bar stress after losses = f_{eff} = 76.3 ksi (OK)

P_{jack} = 109.5(2.60) = 285 kips

Stresses in Concrete Transverse Diaphragms

The post-tensioning rods are located at the centroid of the diaphragms to avoid bending moments. The precast concrete slabs are expected to be at full strength by the time the transverse post-tensioning is applied.

Compressive stress limit: $0.6\phi_w f'_c$ = 0.6(1.0) (5) = 3.00 ksi
(AASHTO Table 5.9.2.3.2a-1)

Maximum compressive stress: $\frac{P_{jack}}{A_g} = \frac{285}{168} = 1.70$ ksi (OK)

Stresses in Longitudinal Joints Between Slabs

Article 5.12.2.3.2 requires the joints between precast slabs be filled with non-shrink grout with a minimum compressive strength of 5.0 ksi at 24 hours. The final stress in the longitudinal joints and transverse diaphragms is estimated as follows.

For area of diaphragms in contact, assume that blockouts will extend to two inches above the bottom of the slab and are eight inches wide.

Total contact area = 4(21-2)8 = 608 in.^{2} (4 diaphragms)

Total area of seven-inch deep shear key = [50(12)-4(8)]7 = 3,976 in.^{2} (exclude diaphragms)

Total contact area = 608 + 3976 = 4,584 in.^{2}

Total prestress force after losses = 4(203.5) = 813.8 kip

Longitudinal joint stress = $\frac{813.8}{4{,}584} = 0.18$ ksi

As previously discussed, Article 5.12.2.3.3c requires a minimum joint stress of 0.25 ksi, however, the basis and application of the stress is not defined. This example uses an alternative method based on the numerical modeling. Actual contact stress is expected to be 28% less than the AASHTO minimum.

Use one 1.75 inch-diameter threaded rod PS bar A722 Gr. 150 at each diaphragm. The transverse post-tensioning ducts are fully grouted after stressing.

NOTATION

a	=	depth of equivalent rectangular compression stress block (in.)
A	=	area of stringer, beam, or component (in.^{2})
A_c	=	concrete area of composite section
A_c	=	concrete area of transformed section
A_{cv}	=	area of concrete engaged in interface shear transfer (in.^{2})
A_{ct}	=	area of concrete on the flexural tension side of member (in.^{2})
A_g	=	gross area of beam section (in.^{2})
A_i	=	area of individual component (in.^{2})
A_0	=	area enclosed by centerline of elements (in.^{2})
A_{ps}	=	area of prestressing steel (in^2)
A_{rect}	=	area of rectangle (in^2)
A_s	=	area of non-prestressed tension reinforcement (in.^{2})
A'_s	=	area of compression reinforcement (in.^{2})
A_{tf}	=	area of transformed section, at final
A_{ti}	=	area of transformed section, at initial
A_v	=	area of transverse reinforcement within distance s (in.^{2})
A_{vf}	=	area of interface shear reinforcing crossing the shear plane within Acv (in.^{2})
A_{void}	=	area of void (in.^{2})
ADL	=	added dead load (kips)
b	=	width of the compression face of a member (in.)
b_{eff}	=	effective flange width (in.)
bL	=	distance from end of beam to harped point (in.)
b_v	=	effective web width taken as the minimum web width, measured parallel to the neutral axis, between resultants of the tensile and compressive forces due to flexure; this value lies within the depth, dv (in.)
b_{vi}	=	interface width (in.)
c	=	distance from extreme compression fiber to the neutral axis (in.) (8.6.11.2); cohesion factor from AASHTO Art 5.8.4.3 (ksi)
cg	=	center of gravity
CGC	=	center of gravity of the concrete section
CGS	=	center of gravity of the strands
D	=	structure depth (ft) or height of standard shape of PC beam given in BDA 6-1

(in.)

DC = weight of supported structures (kip)

d_b = nominal strand diameter (in.)

d_e = effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in.)

DF_{DL} = dead load distribution factor

DF_M = live load moment distribution factor

DF_V = live load shear distribution factor

d_p = distance from extreme compression fiber to the centroid of the prestressing tendons (in.)

d_s = distance from extreme compression fiber to the centroid of the non-prestressed tensile reinforcement (in.)

D_s = superstructure depth (ft)

d_t = distance from extreme compression fiber to the centroid of tensile reinforcement

d_v = the effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (in.)

DW = superimposed dead load (kip)

e = eccentricity of the anchorage device or group of devices with respect to the centroid of the cross section; always taken as positive

e' = difference between eccentricity of prestressing steel at midspan and at span end (in.)

Ec = modulus of elasticity of beam material (ksi)

e_c = eccentricity of strands measured from center of gravity of beam at midspan (in.)

E_{ci} = modulus of elasticity of concrete at initial time (ksi)

E_{ct} = modulus of elasticity of concrete at transfer or time of load application (ksi)

E_{cu} = failure strain of concrete in compression (in./in.)

E_D = modulus of elasticity of deck material (ksi)

e_g = distance between centers of gravity of beam and deck (in.)

e_m = eccentricity at midspan

E_p, E_{ps} = modulus of elasticity of prestressing tendons (ksi)

E_s = modulus of elasticity of mild reinforcing steel (ksi)

e_{tf} = distance between centers of gravity of strands and concrete section at time of service (in.)

e_{ti} = distance between centers of gravity of strands and concrete section at time of transfer (in.)

f_b = concrete flexural stress at extreme bottom fiber (ksi)

f_{bot} = concrete stress at bottom of precast beam (ksi)

f'_c = specified compressive strength of concrete used in design (ksi)

f'_{ci} = specified compressive strength of concrete at time of initial loading or prestressing (ksi); nominal concrete strength at time of application of tendon force (ksi)

f_{cgp} = concrete stress at the center of gravity of prestressing tendons that results from the prestressing force at either transfer or jacking and the self-weight of the member at sections maximum moment (ksi)

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

f_g = stress in the member from dead load (ksi)

f_{pbt} = stress in prestressing steel immediately prior to transfer (ksi)

f_{pe} = effective stress in the prestressing steel after losses (ksi)

f_{pi} = prestressing steel stress immediately prior to transfer (ksi)

f_{pj} = stress in prestressing steel at initial jacking (ksi)

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi)

f_{pu} = specified tensile strength of prestressing steel

f_{py} = yield strength of prestressing steel (ksi)

f_{px} = reduced prestressing steel stress in transfer length (ksi)

f_r = modulus of rupture of concrete (ksi)

f_s = stress in mild tension reinforcement at nominal flexural resistance (ksi)

f'_s = stress in the mild steel compression reinforcement at nominal flexure resistance (ksi)

F_T = required tension capacity provided by reinforcement (kip)

f_{tg} = concrete stress at top of precast beam (ksi)

f_{tgf} = concrete stress at top of precast beam due to Fatigue I combination (ksi)

f_{top} = concrete stress at top of precast beam (ksi)

f_y = yield strength of mild steel (ksi)

f'_y = specified minimum yield strength of compression reinforcement (ksi)

H = average annual ambient mean relative humidity (percent)

h = web dimension of PC beam; depth of slab (in.)

h_{slab} = overall depth of beam (in.)

I = initial gross (non-transformed) moment of inertia of the beam (in.^{4})

I_c = moment of inertia of composite section about centroidal axis, neglecting reinforcement (in.^{4})

I_{cg} = moment of inertia of the beam concrete section about the centroidal axis, neglecting reinforcement (in.^{4})

I_{Ct} = moment of inertia of composite transformed section (in.^{4})

I_e = effective moment of inertia (in.^{4})

I_g = gross moment of inertia of beam (in.^{4})

I_o = moment of inertia of individual component (in.^{4})

I_{rect} = moment of inertia of rectangle (in.^{4})

I_{tf} = moment of inertia of concrete section at final stage, transformed (in.^{4})

I_{ti} = moment of inertia of concrete section at initial stage, transformed (in.^{4})

I_{void} = moment of inertia of void (in.^{4})

J = St. Venant torsional constant (in.^{4})

K_1 = fraction of concrete strength available to resist shear

K_2 = limiting interface shear resistance

K_g = longitudinal stiffness parameter (in.^{4})

K_L = span to depth correction factor (in.^{4})

K_S = skew correction factor (in.^{4})

L = span length or beam length (ft)

LL = live load (kip)

l_{px} = distance from end of beam to d_v from support

M_{cr} = cracking moment (kip-in.)

M_{ADL} = moment due to added dead loads (kip-ft)

M_{DC1} = moment due to self-weight of beam (kip-ft)

M_{DC2} = moment due to self-weight of deck or barrier (kip-ft)

M_{DC3} = moment due to self-weight of barrier (kip-ft)

M_{DW} = moment due to self-weight of future overlay (kip-ft)

M_{dnc} = total unfactored dead load moment action on the monolithic or noncomposite section (kip-ft)

M_f = midspan moment due to fatigue truck (kip-ft)

M_g = midspan moment due to self-weight of beam (kip-ft)

M_{HL93} = moment due to enveloped HL-93 trucks (kip-ft)

M_{LL} = moment due to live loads (kip-ft)

M_n = nominal flexure resistance (kip-in.)

M_{P15} = moment due to enveloped P15 truck (kip-ft)

M_r = factored flexural resistance of a section in bending (kip-in.)

M_{slab} = moment due to weight of deck slab (kip-ft)

M_u = controlling factored moment demand (kip-ft)

M_x = moment at location x (kip-ft)

n = modular ratio between beam and deck

N_b = number of beams, stringers, or beams

N_u = factored axial force, taken as positive if tensile and negative is compressive (kip)

P_e = Effective force in prestressing strands after all losses (kip)

P = Required transverse post-tensioning (kip/ft)

P_c = permanent compressive force (kip)

P_{dia} = transverse diaphragm compressive force (kip)

P_f = force in prestressing strands after losses (kip)

P_{fg} = effective force in prestressing strands after all losses for gross section design (kip)

P_i = force in prestressing strands after elastic shortening loss (kip)

P_j = force in prestressing strands before losses (kip)

P_r = factored bursting resistance of anchorages (kip)

r = radius (in.)

S = spacing of beams, webs, or diaphragms (ft)

S = surface of beam section

s = spacing of reinforcing bars (in.)

s = length of element (in.)

S_b = section modulus for the bottom extreme fiber of the beam where tensile stress is caused by externally applied loads (in.^{3})

S_{BC} = section modulus for the bottom extreme fiber of the composite section where tensile stress is caused by externally applied loads (in.^{3})

S_{BCt} = section modulus for the bottom extreme fiber of the composite section transformed (in.^{3})

S_{Btf} = section modulus for the bottom extreme fiber of the composite section or precast beam - transformed, at service stage (in.^{3})

S_{Bti} = section modulus for the bottom extreme fiber of the composite section or precast beam - transformed, at initial stage (in.^{3})

S_c = section modulus for the extreme fiber of the composite sections where tensile stress is caused by externally applied loads (in.^{3})

S_{nc} = section modulus for the extreme fiber of the monolithic or non-composite sections where tensile stress is caused by externally applied loads (in.^{3})

S_t = section modulus for the top extreme fiber of the sections where tensile stress is caused by externally applied loads (in.^{3})

S_{tc} = section modulus for the top extreme fiber of the composite sections where tensile stress is caused by externally applied loads (in.^{3})

S_{TDCt} = section modulus for the top extreme fiber of the composite sections at top of deck level, at service, transformed (in.^{3})

S_{TGCt} = section modulus for the top extreme fiber of the composite sections at top of beam level, at service, transformed (in.^{3})

S_{Ttf} = section modulus for the top extreme fiber of the composite sections at top of beam level or precast beam, at service, transformed (in.^{3})

S_{Tti} = section modulus for the top extreme fiber precast beam, at initial, transformed (in.^{3})

T = tensile stress in concrete (ksi)

t = thickness of plate like element (in.)

t_d = age of concrete at overlay placement (days)

t_f = age of concrete at final time (days)

t_i = age of concrete at prestress transfer (days)

t_s = thickness of concrete deck slab (in.)

t_h = haunch thickness at midspan (in.)

T_{Hmid} = thickness at midspan (in.)

T_{Hsup} = haunch thickness at support (in.)

V = volume of beam (in.^{3})

V_c = nominal shear resistance provided by tensile stresses in the concrete (kip)

V_n = nominal shear resistance of the section considered (kip)

V_{ni} = nominal interface shear resistance (kip)

V_{ri} = factored interface shear resistance (kip)

V_p = component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip)

V_s = shear resistance provided by the transverse reinforcement at the section under investigation (kip)

V_u = factored shear force (kip)

v_u = average factored shear stress on the concrete (ksi)

V_{ui} = factored interface shear resistance (kip/in.)

W = width of bridge (ft)

w = uniform dead load (kip/ft)

w_{br} = uniform dead load—weight of barrier (kip/ft)

w_c = unit weight of concrete (kcf)

w_{DW} = uniform dead load—weight of overlay (kip/ft)

w_{fw} = uniform dead load—weight of future wearing surface (kip/ft)

w_g = uniform dead load (kip/ft)

w_s = uniform dead load—weight of deck slab (kip/ft)

x = distance from left end of beam (ft)

x_{pA} = influence length of anchor set (ft)

Y = distance from the neutral axis to a point on individual component (in.)

y_b = distance from the neutral axis to the extreme bottom fiber of PC beam (in.)

y_{bts} = centroid of all tensile reinforcement (in.)

y_i = distance from centroid of section *i* to centroid of composite section

y_t = distance from the neutral axis to the extreme top fiber of PC beam (in.)

YTC = distance from the centroid to extreme top fiber of composite section (in.)

Y_{tg} = distance from centroid of the composite section to the extreme top fiber of the PC beam (in.)

Y_{TGCt} = distance from centroid of the composite section to the extreme top fiber of the PC beam (in.)

Y_{Tti} = distance neutral axis to the extreme top fiber of the PC beam, transformed (in.)

α = angle of inclination of transverse reinforcement to longitudinal axis (º) total angular change of prestressing steel path from jacking end to a point under investigation (rad)

α_1 = stress block factor

β = factor relating effect of longitudinal strain on the shear capacity of concrete, as indicated by the ability of diagonally cracked concrete to transmit tension (unitless)

β_1 = ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone (5.7.2.2)

Δ_{Aset} = anchor set length (in.)

Δ_{br} = deflection due to barrier weight (in.)

Δ_g = camber at midspan at erection due to beam self-weight (in.)

$\Delta_{g,erect}$ = camber at midspan at erection due to long-term effects of prestressing force and beam self-weight (in.)

Δ_{ES} = change in length due to elastic shortening

Δ_{fpA} = losses due to anchorage set (ksi)

Δ_{fpCD} = prestress loss due to creep of beam concrete between time of deck placement and final time (ksi)

Δ_{fpCR} = prestress loss due to creep of beam concrete between transfer and deck placement (ksi)

Δ_{fpES} = sum of all losses or gains due to elastic shortening or extension at the time of application of prestress and/or external loads (ksi)

Δ_{fpF} = losses due to friction (ksi)

Δ_{fpLT} = losses due to long-term shrinkage and creep of concrete and relaxation of prestressing steel (ksi)

Δ_{fpR} = an estimation of relaxation loss taken as 2.4 ksi for low relaxation strand, 10 ksi for stress relieved strand, and in accordance with manufacturers recommendation for other types of strand (ksi)

Δ_{fpR1} = prestress loss due to relaxation of prestressing strands between time of transfer and deck placement (ksi)

Δ_{fpR2} = prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time (ksi)

Δ_{fpSD} = prestress loss due to shrinkage of beam concrete between time of deck placement and final time (ksi)

Δ_{fpSR} = prestress loss due to shrinkage of beam concrete between transfer and deck placement (ksi)

Δ_{fpSS} = prestress gain due to shrinkage of deck in composite section (ksi)

Δ_{fpT} = total change in stress due to losses (ksi)

Δ_{fw} = deflection due to future wearing surface (in.)

Δ_g = deflection due to beam self-weight (in.)

Δ_{LL} = deflection due to lane live load (in.)

Δ_{LT} = deflection due to truck live load (in.)

Δ_p = camber at midspan due to prestressing force at release (in.)

Δ_s = instantaneous deflection due to weight of deck slab (in.)

Δ_{dw} = deflection due to overlay (in.)

ε_{cu} = failure strain of concrete in compression (in./in.)

ε_t = net tensile strain in extreme tension steel at nominal resistance (in./in.)

ε_x = longitudinal strain in the web reinforcement on the flexural tension side of the member (in./in.)

θ = angle of inclination of diagonal compressive stresses or skew angle

γ_1 = flexural cracking variability factor

γ_2 = prestress variability factor

γ_3 = ratio of specified minimum yield strength to ultimate tensile strength of reinforcement

γ_h = correction factor for relative humidity of ambient air

γ_{st} = correction factor for specified concrete strength time at of prestress transfer to concrete member

ϕ = resistance factor

μ = coefficient of friction (unitless)

ω = angle of harped strands

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