APPENDIX D – POST-TENSIONING LOSSES AND ELONGATIONS

The following appendix contains all the necessary information and formulas for calculating prestress losses and elongations for prestressed, post-tensioned structures. Included are example calculations for a simple-span structure stressed from one end and for a continuous structure stressed from one end. Also included is an anchor set example calculation.

It should be understood that the formulas and calculations are approximate and the Engineer should apply reasonable tolerances when comparing the actual field measured elongations with those that are theoretical.

Post-Tensioning Losses:

Post-tensioning of prestressed box girder bridges must consider stress losses that will occur. Listed below are seven causes of prestress losses:

1. Friction of the prestressing steel with the duct and loss due to misalignment of the duct.
2. Anchorage slip as the strand wedges seat at the bearing plate.
3. Elastic shortening of the concrete.
4. Creep of the concrete.
5. Shrinkage of the concrete.
6. Relaxation of the prestressing steel.
7. The stressing sequence.

Items 3 to 7 above are losses that take effect after stressing is complete and are assumed to be a total of:

- 20 ksi (138 MPa) for low relaxation wire.
- 22 ksi (152 MPa) for bars.

Items 1 and 2 above are losses that occur during the stressing operation and can be calculated knowing the strand properties and the prestressing tendon path configuration. These are the losses that are of most concern to the Structure Representative.

Friction Loss:

The losses due to friction can be calculated using the following formula:

$$\boxed{T_0 = T_x e^{(\mu\alpha + KL)}} \qquad \textbf{(Equation 1)}$$

where:
- T_o = Steel stress at the jacking end before seating.
- T_x = Steel stress at any point x along tendon path.
- e = Base of Naperian logarithms.
- μ = Friction coefficient.
- α = Total angular change of the prestressing steel profile (tendon path) in radians from the jacking end to a point x.
- K = Wobble coefficient.
- L = Length of prestressing steel from the jacking end to a point x.

The equation $T_o = T_x e^{(\mu\alpha + KL)}$ has been found to overestimate field measurements of elongation for longer frames (greater than 600 feet, or 183 m). In order for Equation 1 to work effectively, values of friction and wobble coefficients for rigid and semi-rigid galvanized metal sheathing have become frame-length dependant, as shown in the following table:

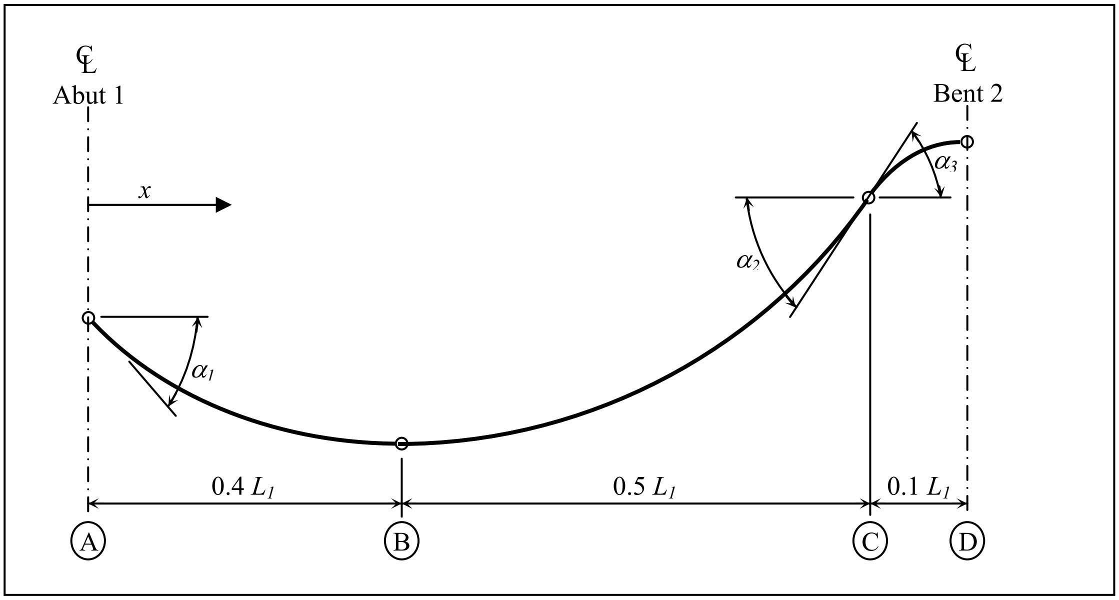
Frame Length (feet)	Wobble Coefficient "K"	Friction Coefficient "μ"
0 - 600	0.0002	0.15
600 - 900	0.0002	0.20
900 - 1200	0.0002	0.25
> 1200	0.0002	See Post-tensioned Technical Committee

The *Standard Specification*[55]*s* requires that the prestress ducts must be rigid and galvanized. Frame length dependant friction and wobble coefficients should be shown in the prestressing notes on the contract plans.

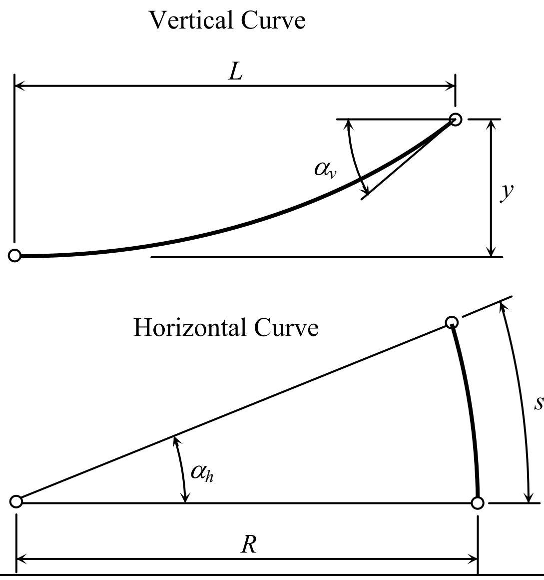
The stress in the prestressing steel at any point "*x*" can be determined by manipulating Equation 1 as follows:

[55] 2010 SS, Section 50-1.02D, *Ducts.*

$$T_x = T_o e^{-(\mu\alpha + KL)}$$ (Equation 2.)



To determine the correction 'α' due to the vertical curvature of the tendon path and for any horizontal bridge curvature that does exist, the following formulas can be used.



$$\alpha_v = 2\frac{y}{L}$$ **(Equation 3)**

$$\alpha_h = \frac{s}{R}$$ **(Equation 4)**

$$\alpha = \sqrt{(\alpha_v)^2 + (\alpha_h)^2}$$ **(Equation 5)**

Where: y = tendon drape in length L
L = length of parabolic curve
s = length of horizontal curve
R = radius of horizontal curve

To determine the loss due to friction expressed as a fraction of the temporary jacking stress, use the following formula:

$$\frac{To - Tx}{To} = 1 - e^{-(\mu\alpha + KL)}$$ **(Equation 6.)**

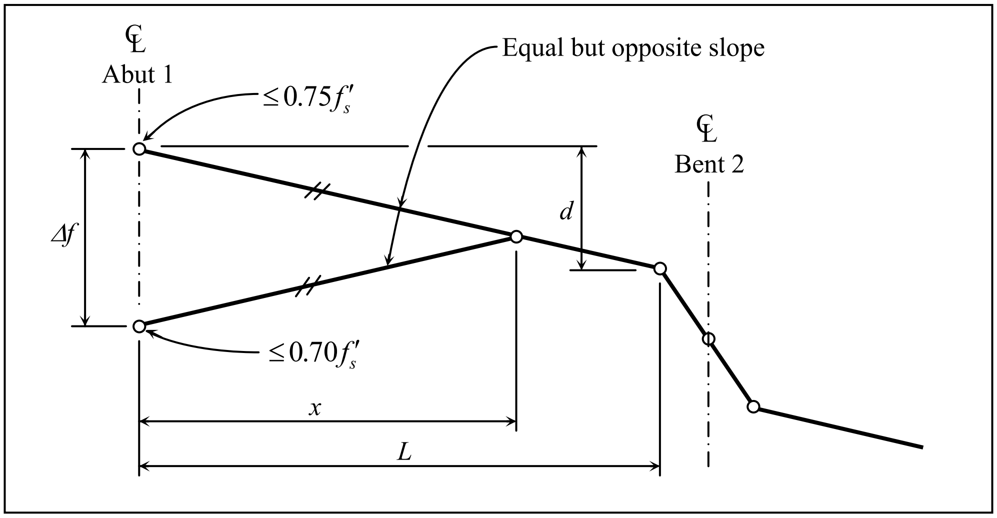
The loss that occurs due to the anchor set can be determined using the following approximate formulas:

$$\Delta f = \frac{2dx}{L}$$ **(Equation 7.)**

$$x = \sqrt{\frac{E(\Delta L)L}{d}}$$ **(Equation 8.)**

where: Δf = change in stress due to anchor set

d	= Friction loss in length L
x	= Length influenced by anchor set
L	= Distance to a point where the loss is known
ΔL	= Anchor set (normally = 3/8″)
E	= Modulus of elasticity, assume 28 x 10 ksi



The *Standard Specifications*[56] requires that the maximum temporary tensile strength (jacking stress before anchor set) must not exceed 75% of the specified minimum ultimate tensile strength of the prestressing steel. This initial stress is just after anchor set but before any long term losses occur, such as concrete shrinkage, relaxation of prestress steel, etc.

[56] 2010 SS, Section 50-1.03B(2)(a), *General.*

Tendon Elongations:

Structure Representatives are responsible for monitoring the Contractor's stressing operations. In addition to the use of a load cell to check prestress force as described earlier in this manual, the strand elongations must be measured and compared with the calculated theoretical elongations.

The Contractor will submit elongation calculations on the shop drawings using assumed values for the modulus of elasticity (E) and the area of the strand (A). When the prestress strand is delivered to the jobsite, it should have an orange release tag with the actual E and A, as determined by METS, written on the back. If these values are not written on the back of this tag, then check the Category 41 file. The E and A should be on the TL-29. In addition, the actual (E) and (A) values determined by the manufacturer for the individual strand packs will also be provided by the Contractor/supplier. The theoretical elongations should be recalculated using the manufacturer's E and A.

The elongation between two points where the stress varies linearly can be given by the following equation:

$$\boxed{\Delta = \frac{T_{avg} L}{E}} \qquad \textbf{(Equation 9.)}$$

where: T_{avg} = Average stress between two points = $(T_1 + T_2)/2$

E = Modulus of elasticity

L = Length between T_1 and T_2

For almost all field situations the elongations based on the numerical average of the end stresses will yield sufficiently accurate results.

Equation 9 above applies to one-end stressing. For two-end simultaneous stressing, the following derivation from Equation 9 can be used.

$$\boxed{\Delta = \frac{T_o (1 + \otimes)(L_1 + L_2)}{2E}} \qquad \textbf{(Equation 10.)}$$

where: $\otimes$ = is the theoretical point of no movement.

The above formulas can be expanded for the entire structure once the theoretical point of no movement or minimum stress is known or calculated. In a continuous structure stressed with two end stressing, the point of no movement in a cable occurs where the losses right of the point equal the losses left of the same point. The force coefficient at that point is shown on the contract plans with the symbol $\otimes$.

If the structure is stressed non-simultaneously, the elongations at the jacking end can be estimated using the assumption that the dead end stress T_e is given by the following formula:

$$\boxed{T_e = T_0(2\otimes - 1)} \qquad \textbf{(Equation 11.)}$$

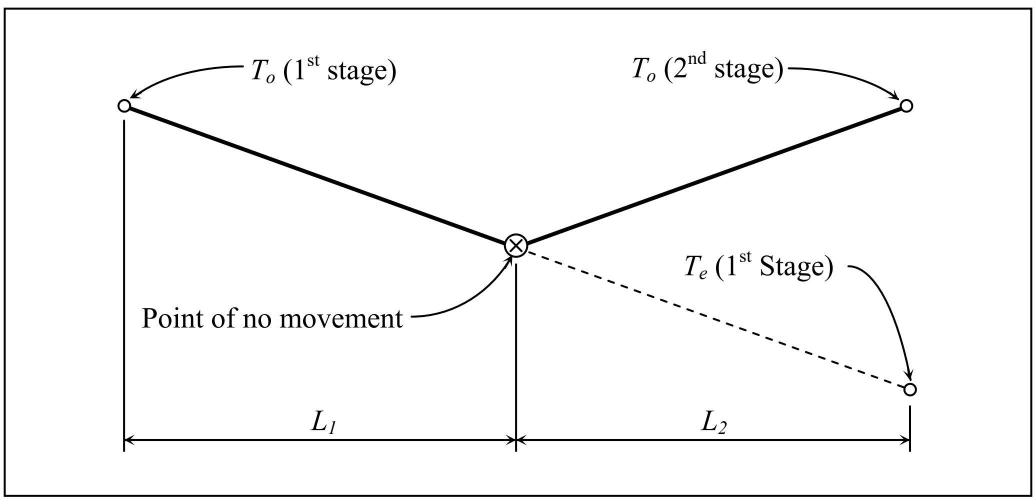
The first and second end elongations are:

$$\boxed{\Delta_{1st} = \frac{T_0}{2E}\left[(1+\otimes)L_1 + (3\otimes - 1)L_2\right]} \qquad \textbf{(Equation 12.)}$$

And:

$$\boxed{\Delta_{2nd} = \frac{T_0(1-\otimes)L_2}{E}} \qquad \textbf{(Equation 13.)}$$

Reasonably accurate elongation calculations can be made for a structure given the following stress diagram:



After obtaining the theoretical elongations, the measurable elongations are calculated. This is usually equal to 80% of the calculated elongation (using the actual E and A) from the first end and 100% from stressing the second end.

In most cases, the use of the ⊗ term as shown on the plans will yield acceptable results. Error is introduced because the calculations are based on a straight-line stress variation and the term is usually an average of tendons and does not account for tendon path length variations.

Checking the tendon length on the shop drawings can be a tedious task, and doesn't warrant accuracy to the ¼ inch. In fact, since elongation varies linearly with tendon length, a tendon length can be off by 1% and not make a significant difference in elongation calculations. For example, if the theoretical elongation for a 300 foot long frame is 24 inches, then a 1% or 3 foot discrepancy in computing the tendon length results in only a 0.24 or ¼ inch difference in elongation.