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In this report we address the problem of cooperative lane change maneuvers where vehicles communicate with each other and negotiate the creation of safe spacings in order to merge without taking any safety risks. The proposed approach requires that the merging vehicle negotiates the creation of a safety gap in the destination lane and till the lane change maneuver is completed it operates as having two possible leaders, one in its own lane and one in the destination lane. In addition, the future following vehicle in the destination lane operates as if the merging vehicle has already changed lanes. This approach leads to a smooth creation of spacings for the vehicle to merge into while safety is guaranteed. Furthermore, we expand this solution to platoons of vehicles, develop the communication protocol to be followed, which includes a rule to avoid possible conflicts, and propose measurement verification steps to identify sensor or communication failures. Extensive simulations are used to demonstrate and evaluate the approach under different conditions. They allow us to verify that the proposed control policy generates the desired smooth gap generation behavior. Moreover, we can conclude that the developed methods improve traffic safety and efficiency and reduce environmental impact.

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# Connected Autonomous Vehicles: Safety During Merging and Lane Change and Impact on Traffic Flow 

## April <br> 2020

A Research Report from the Pacific Southwest Region University Transportation Center

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## Contents

About the Pacific Southwest Region University Transportation Center ..... 4
U.S. Department of Transportation (USDOT) Disclaimer ..... 5
California Department of Transportation (CALTRANS) Disclaimer ..... 5
Disclosure ..... 6
Acknowledgements ..... 7
Abstract ..... 8
Executive Summary ..... 9
1 Introduction ..... 10
2 Literature Review ..... 11
3 Safety Spacing Requirements ..... 12
3.1 Safety Spacing in Vehicle Following ..... 13
3.2 Transient Lane Change Gap ..... 16
3.3 Safety Spacing for Lane Change ..... 17
3.4 Numerical Results ..... 18
4 Lane Changing Protocols ..... 21
4.1 Communications ..... 22
4.2 Conflict Resolution ..... 23
5 Lane Changing and Merging of Platoons ..... 23
5.1 Spacing for Platoon Lane Change ..... 24
5.2 Lane Changing Strategies ..... 25
5.2.1 Synchronous ..... 25
5.2.2 Leader First ..... 25
5.2.3 Last Vehicle First ..... 26
5.3 Numerical Results ..... 27
6 Validation and Verification of Safety Spacing ..... 29
6.1 Information Sources ..... 29
6.2 Verification Protocol ..... 29
6.3 Consistency Check and Failure Identification ..... 30
6.4 Failure Assessment ..... 31
6.4.1 Sensor Failure Mode ..... 31
6.4.2 Communication Failure Modes ..... 32
6.4.3 Risk Assessment Tree ..... 33
7 Minimize Impact on Traffic Flow ..... 33
7.1 Lane Change Phases ..... 34
7.2 Gap Generation Controllers ..... 36
7.2.1 Approach Overview ..... 37
7.2.2 Chosen Vehicle Following Controller ..... 38
7.2.3 Application to Platoons ..... 39
7.3 Lateral Controller ..... 40
8 Simulation Testing and Evaluation ..... 42
8.1 Vehicle Level Simulations ..... 42
8.1.1 Intended Behavior and Safety ..... 42
8.1.2 Metrics ..... 45
8.1.3 Efficiency ..... 46
8.2 Microscopic Simulations ..... 48
8.2.1 Set-up ..... 49
8.2.2 Road-Level Results ..... 50
8.3 Lane-Level Results ..... 51
9 Conclusion ..... 53
10 References ..... 54
11 Data Management Plan ..... 58

## About the Pacific Southwest Region University Transportation Center

The Pacific Southwest Region University Transportation Center (UTC) is the Region 9 University Transportation Center funded under the US Department of Transportation's University Transportation Centers Program. Established in 2016, the Pacific Southwest Region UTC (PSR) is led by the University of Southern California and includes seven partners: Long Beach State University; University of California, Davis; University of California, Irvine; University of California, Los Angeles; University of Hawaii; Northern Arizona University; Pima Community College. The Pacific Southwest Region UTC conducts an integrated, multidisciplinary program of research, education and technology transfer aimed at improving the mobility of people and goods throughout the region. Our program is organized around four themes: 1) technology to address transportation problems and improve mobility; 2) improving mobility for vulnerable populations; 3) Improving resilience and protecting the environment; and 4) managing mobility in high growth areas.

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## Disclosure

Principal Investigator, Co-Principal Investigators, others, conducted this research titled, "Connected Autonomous Vehicles: Safety During Merging and Lane Change and Impact on Traffic Flow" at Ming Hsieh Department of Electrical Engineering, Viterbi School of Engineering, University of Southern California. The research took place from 2/1/2019 to 4/30/2020 and was funded by a grant from the California Department of Transportation in the amount of $\$ 100,000$. The research was conducted as part of the Pacific Southwest Region University Transportation Center research program.

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#### Abstract

Despite the recent advancement of autonomous vehicle technology, performing lane changes in dense traffic environments remains an open challenge. One important issue is finding a suitable space to merge without putting either the lane changing vehicle or others in a situation which they cannot handle in case of an emergency. While humans often put themselves at risk for periods of time, autonomous systems must operate as safely as possible at all times without taking any such risks.

In this report we address the problem of cooperative lane change maneuvers where vehicles communicate with each other and negotiate the creation of safe spacings in order to merge without taking any safety risks. The proposed approach requires that the merging vehicle negotiates the creation of a safety gap in the destination lane and till the lane change maneuver is completed it operates as having two possible leaders, one in its own lane and one in the destination lane. In addition, the future following vehicle in the destination lane operates as if the merging vehicle has already changed lanes. This approach leads to a smooth creation of spacings for the vehicle to merge into while safety is guaranteed. Furthermore, we expand this solution to platoons of vehicles, develop the communication protocol to be followed, which includes a rule to avoid possible conflicts, and propose measurement verification steps to identify sensor or communication failures. Extensive simulations are used to demonstrate and evaluate the approach under different conditions. They allow us to verify that the proposed control policy generates the desired smooth gap generation behavior. Moreover, we can conclude that the developed methods improve traffic safety and efficiency and reduce environmental impact.


# Connected Autonomous Vehicles: Safety During Merging and Lane Change and Impact on Traffic Flow 

Executive Summary

Autonomous vehicles must be able to navigate in vehicle dense environments keeping strict safety guarantees without jeopardizing efficiency. It is particularly hard to obey these objectives during a lane change maneuver, which involves several vehicles and movements in two dimensions. The literature on this subject has already addressed issues spanning from models of human behavior to assessment of maneuver safety and controllers for autonomous lane change. However, procedures to deal with situations where the current state of surrounding vehicles does not satisfy minimum safe conditions are still lacking. We tackle this issue by making use of vehicle connectivity to generate the necessary spacings efficiently. We first investigate safe lane change space definitions and come up with a simplified conservative estimation. This estimate not only takes into account vehicle parameters and current traffic state but is also easily integrated in vehicle following controllers. Then, we address the communication protocol, defining the steps a vehicle must follow from requesting a gap at the desired destination lane until moving into it. Within this framework, we propose using maneuver priority levels to manage conflicts. After that, the use of connectivity leads our focus to maneuvers of vehicle platoons. We not only extend previous results on the minimum safety gaps requirements but also introduce three strategies that allow the whole platoon to perform the maneuver together. They are: Synchronous, Leader First and Last Vehicle First. Next, we turn our attention to another level of safety by studying ways of mitigating possible sensor or communication issues. A verification protocol which compares sensor and communication measurements to internal model predictions is suggested to identify faults. Following a consistency check, an analysis of fault severity determines in which cases the driver is required to take control. After having set the basis for strict safety guarantees, we develop an approach capable of generating the desired behavior in an efficient manner. The vehicle which wants to change lanes adjusts its longitudinal position based both on the preceding vehicle on the same lane and on the vehicle behind which it intends to merge. This is called virtual vehicle following and it ensures safety while positioning the vehicle at the proper place for the maneuver. Simultaneously, the future follower smoothly creates the safe space for the incoming vehicle by adopting the lane changing vehicle as its virtual leader. Besides the theoretical guarantees, we perform extensive simulations to corroborate our claims. A detailed implementation of the controllers in Simulink allows to verify the desired behavior in well-defined scenarios. Moreover, we extract metrics to evaluate the performance of the different platoon strategies under varied traffic conditions. Last, we assess the traffic and environmental impact on a larger scale using the microscopic simulator VISSIM. The results indicate that the proposed methods improve safety and efficiency on highway travels.

## 1 Introduction

The need for more efficient safer vehicles is self-evident: congestion costs were estimated above US $\$ 87$ billion in 2018 [1] and there were almost 35,000 fatalities in 2017 in the United States alone [2]. One promising way of alleviating these issues is through autonomous vehicles [3,4]. While some driving tasks, such as Adaptive Cruise Control (ACC) [5, 6] are already well understood enough to be integrated in commercial vehicles, others still require more study and evaluation. One of these more complicated tasks is lane changing. It is one of the most stressful maneuvers that a driver must perform since it involves changes in longitudinal and lateral velocities while keeping track of relative positions and velocities of several surrounding vehicles. According to the survey in [7], lane changes are responsible for one tenth of all accidents, cause congestion at lane drop locations and act as "moving obstacles", which leads to decreased freeway capacity. Therefore, there is a continuous effort to automate this maneuver.

While there are studies to evaluate lane change feasibility [8-10] and others to determine how to make use of the existing space [11,12], generating the necessary gaps when these do not exist is still an open challenge in the field. A motivating example is the situation of merging into a highway with heavy traffic. In this case, an autonomous vehicle may never find a suitable safe gap. Depending on the highway design, this would force the vehicle to either block all traffic on the access ramp or to stay at the acceleration lane until the next highway exit. While humans drivers can deal with this type of situation by putting themselves at risk, driving algorithms must be designed to avoid prone-to-collision situations. In addition, the advancements in vehicle connectivity give rise to vehicle platoons which are able to increase road throughput thanks to small inter-vehicle following distance $[3,13]$. The side effect noted in [14] is that such small gaps make finding suitable lane change spacings more challenging.

The main goal of this project is to propose solutions that rely on Connected Autonomous Vehicles (CAVs) to guarantee safety and feasibility of lane changes in vehicle dense traffic scenarios. Our efforts are focused on the tactical and operational stages of lane changes, which means we assume the decision to perform the maneuver is given by a higher level decision algorithm. We address the issues of:

1. Feasibility: given the current traffic state, find out if it is safe to start a lane change. If not, determine what is the minimum safe gap;
2. Communication: make use of connectivity to determine feasibility and how to proceed if safe gaps are not available.
3. Platooning: adapt the feasibility checks and the communication strategy when entire platoons of vehicles decide to change lanes;
4. Gap verification: leverage sensors, communication and analytical redundancy to verify the existence of safe gaps;
5. Efficiency: design strategies and controllers that minimize the impact on traffic while guaranteeing safety;
6. Evaluation: demonstrate through simulation the effects of all proposed methods.

After a literature review of the relevant topics in section 2, sections 3 through 8 of this report discuss the above subjects in the given order. Section 9 summarizes the main findings.

## 2 Literature Review

Given the importance of lane change maneuvers highlighted in the Introduction, the continuously growing literature on the subject comes as no surprise. Studies range from attempts to model human behavior [15-17] to controllers capable of performing lane-changes autonomously [11,18]. With a focus on determining maneuver safety, the work in [8] uses a sinusoidal lateral acceleration model alongside constant speed assumptions to compute longitudinal safe gaps between an ego vehicle and the surrounding vehicles before starting the lane change. Then, in [9], situations where there is emergency braking during the lateral movement are studied. These results are used in [19] to analyze safety of platoon maneuvers and determine whether inter-vehicle spacing should increase before all vehicles start lane changing. More recently, works [10] and [20] build upon the results from [8] to propose safe multivehicle lane change planning. A different approach is taken in [21], where, instead of measuring and checking for safe gaps, vehicles consider a maneuver to be safe based on the deceleration they will force on their future follower. Still dealing with maneuver feasibility, perception uncertainty is explicitly included in [22] and [23], whereas [12] uses game theory to estimate other vehicles' reactions and decide when lane changing is safe. From a different perspective, [24] defines a risk level measure to determine the risk of a lane change and uses real world datasets to analyze how human driving relates to that measure. Addressing a related issue, the authors of [25] deal with how to place the vehicle at the appropriate longitudinal position for lane change by solving a quadratic optimization problem with constraints on vehicle position, velocity, acceleration and jerk.

Thanks to standards such as Dedicated Short-Range Communications (DSRC) [26], properly equipped vehicles can communicate either to other vehicles (V2V) or to the infrastructure (V2I) and results from [27] indicate that DSRC's reliability is adequate for communications during lane change. At this point, it is important to differentiate passive from active cooperation. In the former, cooperation is a synonym of communication: the only cooperative action vehicles perform is to share information that others cannot gather from their own sensors. For example, in [18] vehicles use V2V to obtain information about surrounding vehicles and construct artificial potential fields that guide their movement. Differently, in active cooperation, CAVs are expected to alter their behavior to achieve system level goals, such as safety or improved efficiency. Due to the insufficient spacings, lane changing in congested situations requires active cooperation. This idea was first applied in cooperative merging [28], where highway vehicles make space for on-ramp incomers. Despite the success of these methods, they cannot be directly applied to the more general case of lane changes due to the fact that merging must occur within a predefined area. This constraint often yields procedures that depend on the existence of a roadside unit which can communicate to vehicles, perform a centralized optimization and determine the merging order [18]. The main idea we borrow from this domain, as will be seen later, is the one of virtual vehicles [29,30], which al-
lows a vehicle to adjust its longitudinal speed and position based on a vehicle traveling on another lane.

When it comes to creating large enough gaps for lane changes, [31] applies an LQR controller specifically to generate bigger spacings. With another approach, the authors of [32] just change the setpoint of the constant time headway controller already in place and successfully test their algorithm in real vehicles. However, neither approach addresses how to keep the lane changing vehicle at the proper position to initiate the maneuver. This is partially addressed in [33], where VISSIM is used to simulate a scenario in which an accident forces vehicles to change lanes. The paper proposes an algorithm that not only requests vehicles on adjacent lanes to brake in order to create the necessary gaps, but also computes desired speeds for the merging vehicles. While improved throughput in simulations validates the method's usefulness, the gap generating controller works by computing vehicles' speeds, which cannot be directly controlled in practice. Furthermore, there are no proofs that spacings are successfully created for every request. Differently, the work in [34] adopts a nonlinear longitudinal controller derived from well tuned artificial potential fields. When a vehicle has the intention to change lanes, the artificial potential fields in its own controller as well as the ones in the gap generating vehicle are modified in a way which ensures that all vehicles are at safe distances from each other before the maneuver starts. To prove the controllers always yield safe behavior, the authors have to make some restrictive assumptions. First, they consider that the lane changing vehicle has no one in front of it, which is not true in congested scenarios. Second, they expect the future leader to be always longitudinally ahead and faster than the lane changing vehicle. This assumption is violated if the future leader is forced to brake abruptly. In [35] the ACC stems from an optimal control problem whose costs includes factors for safety, efficiency, and comfort. By finding optimal acceleration inputs for different lanes and computing the respective costs, vehicles decide whether to move from their current lane. The paper is not directed at cooperation for gap generation, but this behavior can be observed if vehicles include the costs of others when optimizing their own trajectory. To reduce the computational burden, lane changes can only be performed at predetermined fixed time instants and the optimization problem has no constraints on control inputs or vehicle states. Therefore, to guarantee safety and feasibility of solutions, the cost functions weights must be finely tuned. The demanding task of lane change within platoons is analyzed in [36]. The approach requires that platoon leaders communicate to allow a vehicle to move from one platoon to the other. Given the open-loop nature of the gap generation procedure and the need for platoons to be aligned, unexpected behavior of downstream vehicles might prevent the creation of proper spacings.

## 3 Safety Spacing Requirements

This section discusses how to determine the minimum safe distances between each of the vehicles involved in the lane change maneuver. To do that, the ego vehicle $E$, which is planning to perform the maneuver, must consider a few factors. The first one is that a safe vehicle following distance to vehicles on the same lane must be maintained till the lane change maneuver is completed. The second is a transient gap, which takes care of the relative distance variation during the maneuver


Figure 1: In (a), minimum safe lane change gaps that must be created before the maneuver can start. In (b), expected distances right after the maneuver is completed.
while considering both longitudinal and lateral positions. The final factor is that, during the lane change, $E$ 's braking capability decreases. We also note that $E$ must guarantee large enough distances to three of its surrounding vehicles, namely: leader in the destination lane ( $L_{d}$ ), follower in the destination lane $\left(F_{d}\right)$, and leader in the original lane $\left(L_{o}\right)$. The distance to the follower in the original lane $\left(F_{o}\right)$ is not addressed, since this vehicle is assumed have its own safe vehicle following controller. These ideas are summarized by Fig. 1, where $g_{i, j}^{\mathrm{f}}(t)$ is the safe vehicle following distance between vehicles $i$ and $j$ at time $t$ and $\Delta g_{i, j}(t)$ is the transient gap. The rest of this section details how to compute the safe spacings and presents numerical results of the proposed methods.

### 3.1. Safety Spacing in Vehicle Following

Safe vehicle following can be defined as keeping a sufficient distance from a leading vehicle that is in same lane as the ego vehicle such that, if the lead vehicle stops in an emergency, the ego vehicle can also do so without any collision. To compute the minimum safe distance, let's start by defining the gap between the ego vehicle $E$ and its leader $L$ :

$$
\begin{equation*}
g_{E, L}(t)=\left(x_{L}(t)-l_{L}\right)-x_{E}(t), \tag{1}
\end{equation*}
$$

where $l_{i}$ is the length of vehicle $i, i \in\{E, L\}$, and $x_{i}(t)$ is the longitudinal position of its front bumper at time $t$. The gap evolves over time as:

$$
\begin{equation*}
g_{E, L}(t)=g_{E, L}\left(t_{0}\right)+\left(v_{L}\left(t_{0}\right)-v_{E}\left(t_{0}\right)\right) t+\int_{t_{0}}^{t} \int_{t_{0}}^{\lambda}\left(a_{L}(\tau)-a_{E}(\tau)\right) \mathrm{d} \tau \mathrm{~d} \lambda \tag{2}
\end{equation*}
$$

where $v_{i}(t)$ and $a_{i}(t)$ are vehicle $i$ 's longitudinal velocity and acceleration respectively, and $t_{0}$ is the initial time. To guarantee safety in an emergency braking scenario, we need $g_{E, L}(t) \geq 0$ for


Figure 2: Acceleration profiles for the leader and follower.
$t_{0} \leq t \leq T$, where $t_{0}$ takes the value of the time instant when the leader starts braking and $T$ is the time when both vehicles achieve full stop. Therefore, the minimum safe gap is:

$$
\begin{align*}
& g_{E, L}\left(t_{0}\right) \geq\left(v_{E}\left(t_{0}\right)-v_{L}\left(t_{0}\right)\right) t+\int_{t_{0}}^{t} \int_{t_{0}}^{\lambda}\left(a_{E}(\tau)-a_{L}(\tau)\right) \mathrm{d} \tau \mathrm{~d} \lambda, \quad \forall t \in \mathbb{T} \\
& g_{E, L}^{*}\left(t_{0}\right)=\max _{t \in \mathbb{T}}\left[\left(v_{E}\left(t_{0}\right)-v_{L}\left(t_{0}\right)\right) t++\int_{t_{0}}^{t} \int_{t_{0}}^{\lambda}\left(a_{E}(\tau)-a_{L}(\tau)\right) \mathrm{d} \tau \mathrm{~d} \lambda\right], \tag{3}
\end{align*}
$$

where $\mathbb{T}=\left[t_{0}, T\right]$. To compute values for $g_{E, L}^{*}\left(t_{0}\right)$, we need to define the leader's emergency braking behavior and $E$ 's reaction to that. Concerning the former, the most conservative case, known as "brick wall" [37,38], assumes that the leader stops instantaneously. This assumption leads to impractically large distances. In [37] and [6], the leader is assumed to have zero acceleration at $t_{0}$, apply maximum negative jerk until reaching maximum deceleration $-D_{L}$ and then keep this deceleration until full stop. The ego vehicle is assumed to have maximum acceleration $A_{E}$ at $t_{0}$. This can be viewed as a situation where $E$ is trying to "catch-up" with $L$. After a delay $\tau_{1}, E$ realizes its leader is braking. However, at first, the ego vehicle does not know this is an emergency situation and applies a comfortable negative jerk $-J_{E, c}$ until it reaches a comfortable deceleration $-D_{E, c}$. After a second delay $\tau_{2}$, $E$ notices that this is an emergency scenario. By then, it brakes with maximum jerk $-J_{E}$ until it reaches a maximum deceleration $-D_{E}$. Finally, it keeps this deceleration until full stop. The whole process is summarized by Fig. 2. The authors of [6] vary the delay values $\tau_{1}$ and $\tau_{2}$ to model three classes of drivers: human, autonomous and connected. The first has both delay values larger than the second, while the connected vehicle skips the comfortable


Figure 3: Severity as a function of the initial gap for the different driver types described in [6].
braking phase and goes straight into emergency braking after a single delay.
If, $g_{E, L}\left(t_{0}\right)<g_{E, L}^{*}\left(t_{0}\right)$, collision severity can be estimated by a measure called Delta-V [39], which describes the absolute difference in vehicle speed before and after a crash. Under the assumption of inelastic collision:

$$
\begin{equation*}
\Delta v_{E}=\frac{m_{L}}{m_{E}+m_{L}}\left(v_{E}\left(t_{-}\right)-v_{L}\left(t_{-}\right)\right) \tag{4}
\end{equation*}
$$

where $m_{i}$ is the mass of vehicle $i$ and $v_{i}\left(t_{-}\right)$is the velocity of vehicle $i$ right before collision. Assuming vehicles of equal mass, severity is plotted against initial gap for the three classes of driver in Fig. 3. The advantages of CAVs become evident: it is possible to have considerably smaller inter vehicle gaps, which increase road capacity, without compromising safety.

In what follows, we assume that vehicles are connected, so $E$ goes straight into emergency braking after a delay $\tau$, and that $D_{L} \geq D_{E}$, which means the leader is able to brake at least as hard as the ego vehicle. This yields the closed form solution for Eq. (3):

$$
\begin{equation*}
g_{E, L}^{*}\left(t_{0}\right)=\frac{v_{E}^{2}\left(t_{0}\right)}{2 D_{E}}-\frac{v_{L}^{2}\left(t_{0}\right)}{2 D_{L}}+\lambda_{1} v_{E}+\lambda_{2} \tag{5}
\end{equation*}
$$

where:

$$
\begin{equation*}
\lambda_{1}=\tau+\frac{A_{E}+D_{E}}{J_{E}}+\frac{1}{D_{E}}\left(A_{E} \tau+\frac{A_{E}\left(A_{E}+D_{E}\right)}{J_{E}}-\frac{\left(A_{E}+D_{E}\right)^{2}}{2 J_{E}}\right) \tag{6}
\end{equation*}
$$

and

$$
\begin{align*}
\lambda_{2}= & \frac{1}{2} A_{E} \tau^{2}+\frac{A_{E}\left(A_{E}+D_{E}\right)^{2}}{2 J_{E}^{2}}-\frac{\left(A_{E}+D_{E}\right)^{3}}{6 J_{E}^{2}}+\frac{A_{E}\left(A_{E}+D_{E}\right)}{J_{E}} \tau+  \tag{7}\\
& \frac{1}{2 D_{E}}\left(A_{E} \tau+\frac{A_{E}\left(A_{E}+D_{E}\right)}{J_{E}}-\frac{\left(A_{E}+D_{E}\right)^{2}}{2 J_{E}}\right)^{2} .
\end{align*}
$$

Variables $\lambda_{1}$ and $\lambda_{2}$ are measured in seconds and meters respectively and their computation is detailed in [5]. Setting $D_{E}=\gamma D_{L}$, where $0<\gamma \leq 1$, describes the more limited braking of the ego vehicle. Trucks following passenger vehicles, for example, should adopt low values of $\gamma$. Moreover, let $v_{L}\left(t_{0}\right)=\rho v_{E}\left(t_{0}\right)$, where $0<\rho \leq 1$, in order to account for possible velocity differences. This yields:

$$
\begin{equation*}
g_{E, L}^{*}\left(t_{0}\right)=\left(\frac{1-\gamma \rho^{2}}{2 D_{E}} v_{E}\left(t_{0}\right)+\lambda_{1}\right) v_{E}\left(t_{0}\right)+\lambda_{2} \tag{8}
\end{equation*}
$$

It should be noted that setting either $\gamma$ or $\rho$ equal to zero is equivalent to the "brick wall" scenario. We can define:

$$
\begin{equation*}
h=\frac{1-\gamma \rho^{2}}{2 D_{E}} \bar{v}_{E}+\lambda_{1} \tag{9}
\end{equation*}
$$

where $\bar{v}_{E}$ is an upper limit on $E$ 's velocity, such as the road's maximum speed or the desired traveling velocity. While the relationship of $h$ to $D_{E}$ remains non-linear due to $\lambda_{1}$, the sensitivity with regard to the difference in braking capabilities and initial speeds is clear. This leads to a safe vehicle following distance known as constant time headway policy, which is seen often in the Adaptive Cruise Control Literature $[5,40]$ and is used throughout this work:

$$
\begin{equation*}
g_{E, L}^{\mathrm{f}}\left(t_{0}\right)=h v_{E}\left(t_{0}\right)+d_{0}, \tag{10}
\end{equation*}
$$

where $h$ is usually called the time headway, and $d_{0}$ is the desired distance at standstill. For simplicity, we choose $d_{0}=\lambda_{2}$.

### 3.2. Transient Lane Change Gap

Before a lane change, velocities of $E, F_{d}$ and $L_{d}$ can differ considerably and it is desirable to avoid large acceleration or deceleration while performing the lateral adjustment. Therefore, it is necessary to take into account the longitudinal distance variation during the maneuver. This ensures that, despite non-zero relative velocities between $E$ and the surrounding vehicles, the lane change starts and ends respecting safe vehicle following distances. To achieve this, we follow the approach from [8]. First, let $E$ 's lateral acceleration be modeled as:

$$
a_{y}(t)= \begin{cases}\frac{2 \pi H}{\Delta t_{\mathrm{lc}}^{2}} \sin \left(\frac{2 \pi}{\Delta t_{\mathrm{lc}}}\left(t-t_{\mathrm{adj}}\right)\right), & t_{\mathrm{adj}} \leq t \leq t_{\mathrm{lc}}  \tag{11}\\ 0, & \text { otherwise }\end{cases}
$$

where $H$ is the total lateral displacement, $t_{\text {adj }}$ is the time instant after longitudinal adjustments, $t_{l c}$ is the time instant after the lane change is completed and $\Delta t_{\mathrm{lc}}=t_{\mathrm{lc}}-t_{\mathrm{adj}}$ is the lane change duration. The equation above is used to determine the time intervals $\mathbb{T}_{k}=\left[t_{k}, T_{k}\right]$ in which the lateral coordinates indicate that a collision between $E$ and $k \in\left\{L_{d}, F_{d}, L_{o}\right\}$ is possible. The obtained solution for relative distance variation during lane change is similar to Eq. (3) changing $\mathbb{T}$ by $\mathbb{T}_{k}$.

Because we demand safe vehicle following distances to be respected at the start and end of the lane change, we only need to consider $t_{k} \geq t_{\text {adj }}$ and $T_{k} \leq t_{\mathrm{lc}}, \forall k$. Furthermore, we assume all vehicles keep constant speed during lane change. The method to guarantee this strong assumption holds is presented further on in section 7. Thus, the gap variation during lane change to the leaders is:

$$
\begin{equation*}
\Delta g_{E, k}\left(t_{\mathrm{adj}}\right)=\max _{t \in \mathbb{T}_{k}}\left[\left(v_{E}\left(t_{\mathrm{adj}}\right)-v_{k}\left(t_{\mathrm{adj}}\right)\right) t\right] \tag{12}
\end{equation*}
$$

where $k=\left\{L_{o}, L_{d}\right\}$ and to the future follower is:

$$
\begin{equation*}
\Delta g_{F_{d}, E}\left(t_{\mathrm{adj}}\right)=\max _{t \in \mathbb{T}_{F_{d}}}\left[\left(v_{F_{d}}\left(t_{\mathrm{adj}}\right)-v_{E}\left(t_{\mathrm{adj}}\right)\right) t\right] . \tag{13}
\end{equation*}
$$

### 3.3. Safety Spacing for Lane Change

Finally, one must account for the possibility of braking during the lane change. It is well known that, when the vehicle's wheels form a non-zero angle with its moving direction, its braking capabilities are reduced. This idea, expressed by the friction circle [40], can be described by

$$
\begin{equation*}
a_{x}^{2}+a_{y}^{2}=C \tag{14}
\end{equation*}
$$

where $a_{x}$ and $a_{y}$ are the vehicle's longitudinal and lateral acceleration respectively, and $C$ is a positive constant related to tire friction. It should be noted that the above equation is only valid when braking happens during lateral movement. For pure longitudinal braking the maximum longitudinal acceleration is not constrained to $\sqrt{C}$. The authors of [9] use this equation to develop an algorithm that computes safe distances by considering all possible initial braking times. We adopt a simpler, more conservative approach which allows the vehicle to keep using a time headway based distance. It is derived as follows. Starting from Eq. (11), it is possible to obtain the maximum lateral acceleration $A_{y}=2 \pi H / \Delta t_{\mathrm{lc}}^{2}$, use it in the friction circle equation and compute the minimum maximum longitudinal braking during lane change:

$$
\begin{equation*}
D_{E, \mathrm{lc}}^{2}=C-A_{y}^{2} \tag{15}
\end{equation*}
$$

By construction, $D_{E, \text { lc }}<D_{E}$, which yields larger values for the safety vehicle following spacing. To get the values of $h_{\mathrm{lc}}$ and $d_{0, \mathrm{lc}}$, we use both $D_{E, \mathrm{lc}}$ and a new value $A_{E, \mathrm{lc}}$ based on the expected acceleration at the beginning of the maneuver. This implies $E$ has to increase its distance to $L_{o}$ before starting the maneuver. Finally, the safe gaps for lane changing are a sum of the safe vehicle

Table 1: Vehicle parameters.

| Parameter [unit] | Passenger Vehicle | Truck |
| :---: | :---: | :---: |
| mass $[\mathrm{kg}]$ | 2000 | 18000 |
| length $[\mathrm{m}]$ | 5 | 18 |
| width $[\mathrm{m}]$ | 1.8 | 2.4 |
| $A_{\max }\left[\mathrm{m} / \mathrm{s}^{2}\right]$ | 4 | 2 |
| $-D_{\max }\left[\mathrm{m} / \mathrm{s}^{2}\right]$ | -8 | -3 |
| $\left\|J_{\max }\right\|\left[\mathrm{m} / \mathrm{s}^{3}\right]$ | 50 | 30 |
| $\delta[\mathrm{~s}]$ | 0.3 | 0.3 |

following distance and the transient gaps:

$$
\begin{align*}
& g_{E, L_{o}}^{\text {lc }}\left(t_{\mathrm{adj}}\right)=g_{E_{l c}, L_{o}}^{\mathrm{f}}\left(t_{\mathrm{adj}}\right)+\Delta g_{E, L_{o}}\left(t_{\mathrm{adj}}\right),  \tag{16}\\
& g_{E, L_{d}}^{\mathrm{lc}}\left(t_{\mathrm{adj}}\right)=g_{E_{l c}, L_{d}}^{\mathrm{f}}\left(t_{\mathrm{lc}}\right)+\Delta g_{E, L_{d}}\left(t_{\mathrm{adj}}\right),  \tag{17}\\
& g_{F_{d}, E}^{\mathrm{lc}}\left(t_{\text {adj }}\right)=g_{F_{d}, E}^{\mathrm{f}}\left(t_{\text {lc }}\right)+\Delta g_{F_{d}, E}\left(t_{\mathrm{adj}}\right), \tag{18}
\end{align*}
$$

where $E_{l c}$ highlights the fact that $D_{E, \mathrm{lc}}$ is used in the computations. The safe vehicle following distances to vehicles at the destination lane are computed using the time $t=t_{\mathrm{lc}}$, because the gaps between them and $E$ must be safe by the time the maneuver is completed. Once the lane change is done, $E$ can revert back to using $D_{E}$ in its computations. The total necessary gap at the destination lane is:

$$
\begin{equation*}
g_{\mathrm{dest}}\left(t_{\mathrm{adj}}\right)=g_{E, L_{d}}^{\mathrm{lc}}\left(t_{\mathrm{adj}}\right)+l_{E}+g_{F_{d}, E}^{\mathrm{lc}}\left(t_{\mathrm{adj}}\right) \tag{19}
\end{equation*}
$$

### 3.4. Numerical Results

The proposed methods are evaluated in simulation with realistic values. Two vehicle types are used: passenger vehicle and trucks. Their parameters are detailed in table 1. It should be noted that some of these parameters depend on the current speed, but they are set constant to ease visualization and analysis.

Let's start by analyzing how safe vehicle following spacings vary with relative speed and different vehicle types. In Figure 4, each plot represents an ego vehicle/ leader combination: passenger vehicle/ passenger vehicle (PP), passenger vehicle/ truck (PT), truck/ passenger vehicle (TP), and truck/ truck (TT). The red lines represent the vehicle following gap from Eq. (10) and the color map indicates collision severity for the vehicle of lowest mass. In all cases, the leader's initial velocity is $v_{L}\left(t_{0}\right)=90 \mathrm{~km} / \mathrm{h}$ and, when computing the time headway, the ego vehicle assumes $\rho=0.9$, and $\bar{v}_{E}=108 \mathrm{~km} / \mathrm{h}$. Moreover, if both vehicles are of the same type, we assume the ego vehicle to brake at most with $\gamma D_{\max }$, where $\gamma=0.85$. The computed vehicle following gap is an overestimation of the non-linear collision free boundary as long as all initial assumptions on maximum decelerations and velocities hold. For the three cases where the follower cannot brake as


Figure 4: The red line represents the minimum safe vehicle following distance as a function of the relative velocity The color map indicates the severity, as computed from Eq. (4), of a collision under the described worst-case scenario for each combination of initial gap and velocity.

Table 2: Parameters for Lane Change Spacing.

| Parameter [unit] | Value |
| :---: | :---: |
| $H[m]$ | 3.6 |
| $\Delta t_{\mathrm{lc}}[s]$ | 5 |
| $t_{\text {adj }}[s]$ | 0 |
| $\gamma_{F_{d}}$ | 0.85 |
| $\gamma_{L_{d}}, \gamma_{L_{o}}$ | 1.15 |
| $\gamma_{E}$ | 1 |
| $-D_{\mathrm{LC}}\left[\mathrm{m} / \mathrm{s}^{2}\right]$ | -4 |
| $A_{\mathrm{LC}}\left[\mathrm{m} / \mathrm{s}^{2}\right]$ | 0 |



Figure 5: Minimum lane change safe spacing from $L_{o}$ (blue), $F_{d}$ (red) and $L_{d}$ (yellow) as a function of relative velocity.
hard as its leader (Figs. $4 \mathrm{a}, 4 \mathrm{c}$ and 4 d ), the gaps are conservative while $v_{E}(0) \leq \min \left[v_{L}(0) / \rho, \bar{v}_{E}\right]$. A well designed vehicle following controller can keep the vehicle within these limits. In the case when the follower has a greater braking capability than its leader, the assumption for the closed form solution of Eq. (5) is no longer respected and, thus, the computed safe gap is conservative for the whole relative velocity range. Comparing the four plots, it is clear that trucks need larger spaces ahead of them than passenger vehicles and that mass differences lead to different maximum severities.

Next, safe lane change spacings are computed. The scenario includes four passenger vehicles with maximum deceleration $D_{k}=\gamma_{k} D_{\max }$, where $k=\left\{L_{o}, L_{d}, E, F_{d}\right\}$. The values of $\rho$ and $\bar{v}$ are as in the previous experiment. Table 2 provides values for $\gamma_{k}$ 's along with the other required parameters. Figure 5 shows the minimum gaps to each vehicle as a function of relative speeds when all but the ego are traveling at $90 \mathrm{~km} / \mathrm{h}$. The results agree with intuition: the faster the ego vehicle is traveling, the farther away from its leaders in the original ( $L_{o}$ ) and destination lane ( $L_{d}$ ) it must be. The converse is true regarding its follower in the destination lane ( $F_{d}$ ). Moreover, we


Figure 6: Risk map for the red ego vehicle. All vehicles are moving from left to right and the color map represents collision severity for the worst case scenario. On top, results considering only vehicle following. On the bottom, the risk map when lane change factors are taken into account. The yellow rectangle shows the expected relative distance variation during the maneuver.
see how safe distances increase more sharply once the ego vehicle is faster than its leaders. This effect is due to non-linearities in the transient lane change gap.

Last, these results can be used to create a risk map of the ego vehicle's surroundings. Let there be a scenario with parameters as before: followers have less and leaders have more braking capability then $E$. Moreover, vehicles in the origin lane are all traveling at $90 \mathrm{~km} / \mathrm{h}$ and vehicles at the destination lane are all traveling at $100 \mathrm{~km} / \mathrm{h}$. For comparison, a risk map that considers only safe vehicle following is shown at the top of Figure 6. It is clear that the red ego vehicle is at a safe position in its original lane if there is no intention for lane changes. The bottom of the same figure presents a risk map which explicitly takes lane change into account. The rectangle with yellow edges shows $\Delta g_{F_{d}, E}(t)$, the gap variation between $F_{d}$ and $E$ during a lane change. Therefore, the accident prone area in front of $F_{d}$ is shifted forward to represent the situation at the end of a possible lane change. Another visible difference is the increase of the risk areas behind both leaders. This is a consequence of $E$ 's reduced maximum braking during lateral movement. This risk map helps to show how finding safe lane change spacings is challenging and why the communication between vehicles is necessary.

## 4 Lane Changing Protocols

In this section, we present the communication protocol to be followed by vehicles involved in the lane change. We define which parameters must be exchanged by vehicles, how the communication should occur to allow for gap generation requests and how to deal with conflicts.


Figure 7: Communication protocol for lane change.

### 4.1. Communications

Safety applications built to operate on connected scenarios commonly use Basic Safety Message (BSM) packets, which are sent at a 10 Hz frequency and whose typical communication range is around $500 \mathrm{~m}[27,41]$. These packets contain data such as vehicle size, position, speed, and acceleration. To compute the lane change safe gaps, $E$ requests, in addition: mass, width, and maximum acceleration, deceleration and jerk of its surrounding vehicles. Furthermore, if the gap on the destination lane is too small, $E$ can request $F_{d}$ to lower its speed and create a big enough space. This procedure has two benefits. First, it improves maneuver safety, since no one has to force itself on a risky position at the destination lane. Second, it prevents vehicles from being stuck on a lane. This is particularly detrimental for vehicles trying to move towards a highway exit or trying to leave a merging lane. Moreover, having such a procedure is fundamental for platoons, since they demand more space to merge as is shown further on.

The communications to set up the maneuver must follow some predefined protocol. An overview of our proposed method is presented in Fig. 7. The details for each step are:
Ego vehicle has intention to change lanes: it is assumed that a higher level algorithm determines not only whether the vehicle wants to change lanes, but also why. The reason to perform a maneuver is used further on for conflict resolution, i.e., when several vehicles are trying to change lanes in the same region.
Ego vehicle sends information request and priority level of its maneuver: the ego vehicle has to modify its next message sent through DSRC to include a request for more information about those within its range. In addition, $E$ also informs others about its maneuver priority, which depends directly on the reason for the intention to change lanes. The different priorities are described later on together with the conflict resolution rules.
Contacted vehicles transmit relevant data to the ego vehicle: each vehicle sends two sets of data to $E$. The first one contains the vehicle's own parameters: length, mass, width, and maximum acceleration, deceleration and jerk, as well as their current speed and acceleration. The second set informs whether the contacted vehicle is willing to cooperate with the maneuver and whether there is a conflict.
Ego vehicle determines if, when and where to merge: having all the necessary data, the ego vehicle checks whether it can merge under the current traffic state. If necessary, it might wait for
other maneuvers to end. In low density situations, the ego vehicle can opt to perform lane change with non-zero relative speed to vehicles in the adjacent lane, and to adjust its velocity afterwards. However, under congested scenarios, the current gaps are not large enough and $E$ has to choose which vehicle will generate the proper spacing.
Ego vehicle informs others of its decision: $E$ uses the vehicles' unique identifiers to specify which one should become its follower on the destination lane and it shares its own parameters with this vehicle. This allows $F_{d}$ to know the size of the gap to generate for $E$. Moreover, all vehicles within range are aware that a lane change is taking place and can monitor for conflicts.
Vehicles perform longitudinal adjustments: $F_{d}$ adjusts its distance to $L_{d}$ until the gap is big enough for the ego vehicle to merge. At the same time, $E$ must ensure it does not overtake $L_{d}$, while keeping a safe distance to $L_{o}$. The controller that achieves these objectives is presented further on in this report.
If gap is safe, the ego vehicle starts the lane change: the ego vehicle continuously monitors the three relevant spacings with its onboard sensors and starts the maneuver when they meet the safety requirements.

### 4.2. Conflict Resolution

When the ego vehicle starts communicating with others, it informs its lane change priority level as:

1. Urgent: lane change to avoid high risk situation;
2. High: mandatory lane change, such as merging;
3. Medium: lane change towards a route goal, such as reaching an exit;
4. Low: discretionary lane change to achieve higher speed.

To deal with possible conflicts, any vehicle which receives a request keeps the priority information stored until the maneuver is finished or up to a certain expiration time limit. If this vehicle receives another request, it identifies a possible conflict. If two requests have the same priority, they are treated in a first come, first served basis. If a higher priority request arrives to a certain vehicle, it is this vehicle's responsibility to determine whether the maneuver requested earlier should be delayed and then perform the proper communication. Since each lane changing vehicle may communicate to several others in the adjacent lane, we can define a minimum number of vehicles between two possible new gaps. In other words, if the lane changes are longitudinally far from each other, they may happen simultaneously.

## 5 Lane Changing and Merging of Platoons

This section extends previous results of single-vehicle lane change to vehicle platoons. A platoon is a group of vehicles with similar characteristics that use communication to follow each other closely
and safely. Platoons are shown to improve road utilization in works such as [3]. Moreover, truck platoons are motivated both from the lack of truck drivers and from drag reduction which improves fuel economy. The study of platoon lane changes is therefore important to avoid the situation where each vehicle negotiates its maneuver independently. The free-for-all approach has at least two clear disadvantages when compared to a unified platoon strategy. First, in the former, there would be several vehicles on the original lane trying to communicate to vehicles at the destination lane at the same time, while, in the latter, a single platoon vehicle can establish communication and negotiate for the whole platoon. Second, if each one does their maneuver individually, there will no longer be a platoon after the lane change. In what follows, we first determine the total space which has to be generated at the destination lane for a platoon to safely fit. This step helps to quantify the disturbance created in the destination lane. Then, we present three platoon lane changing strategies which describe how the platoon vehicles can use the gap being generated for them.

### 5.1. Spacing for Platoon Lane Change

Let's start by analyzing some of the platoon's characteristics. When it is at steady-state:

$$
\begin{align*}
v_{n}(t) & =v_{P}(t) & & \forall n, \\
g_{n, n-1}(t) & =h_{n} v_{P}(t)+d_{0, n}, & & \forall n, \tag{20}
\end{align*}
$$

where $v_{n}$ is the speed of vehicle $p_{n}$ and $v_{P}$ is the platoon speed. The index $n \in[1, N]$ represents the vehicle's position in the platoon, with 1 being the platoon leader and $N$ being the last vehicle. These assumptions yield a platoon length of

$$
\begin{equation*}
l_{P}(N, t)=l_{1}+\sum_{n=2}^{N}\left(h_{n} v_{P}(t)+d_{0, n}+l_{n}\right) \tag{21}
\end{equation*}
$$

where $l_{n}$ is the vehicle's length. If vehicles are uniform, we define $l_{n}=l, h_{n}=h$ and $d_{0, n}=d_{0}$, $\forall n$, and the above formula is simplified to:

$$
\begin{equation*}
l_{P}(N, t)=N l+(N-1)\left(h v_{P}(t)+d_{0}\right) \tag{22}
\end{equation*}
$$

We note that the platoon length grows as a function of the number of vehicles $N$, the traveling velocity $v_{P}(t)$, and vehicles' characteristics $l, h$ and $d_{0}$. Extending the single vehicle result to platoons, the necessary gap at the adjacent lane is:

$$
\begin{equation*}
g_{\mathrm{dest}}\left(t_{\mathrm{adj}}\right)=g_{p_{1}, L_{d}}^{\mathrm{lc}}\left(t_{\mathrm{adj}}\right)+l_{P, \mathrm{lc}}\left(N, t_{\mathrm{adj}}\right)+g_{F_{d}, p_{N}}^{\mathrm{lc}}\left(t_{\mathrm{adj}}\right), \tag{23}
\end{equation*}
$$

where $l_{P, \mathrm{lc}}\left(N, t_{\mathrm{adj}}\right)$ is the length of the platoon after vehicles update their braking capabilities from $D_{p}$ to $D_{p, \mathrm{lc}}$. By construction, $l_{P, \mathrm{lc}}\left(N, t_{\mathrm{adj}}\right)>l_{P}\left(N, t_{\mathrm{adj}}\right)$, which means that platoon vehicles must increase their distances to each other before starting the maneuver. After completing the maneuver, they can go back to the original inter-vehicle gaps. The first and last terms of the necessary gap


Figure 8: The platoon of red vehicles waits for a proper spacing on the adjacent lane before they can start a synchronized maneuver.
are similar to the single vehicle case, with $E$ being substituted by $p_{1}$ and $p_{N}$ respectively. Thus, the platoon length quantifies how much more disturbance is caused in the destination lane due to a joint platoon maneuver.

### 5.2. Lane Changing Strategies

The platoon vehicles can use the gap being created at the destination lane in different ways. In unexpected cases, such as $L_{d}$ or $L_{o}$ performing emergency braking after the platoon has started changing lanes, the joint maneuver is aborted and the platoon is split in two: one formed by vehicles already in the destination lane and one by vehicles still in the origin lane.

### 5.2.1. Synchronous

The first strategy is to have the all platoon vehicles performing the maneuver simultaneously. This requires that gaps from $p_{1}$ to $L_{o}$ and $L_{d}$ and from $F_{d}$ to $p_{N}$ respect the safety conditions:

$$
\begin{equation*}
g_{i, j}(t) \geq g_{i, j}^{\text {lc }}(t) \tag{24}
\end{equation*}
$$

where the pairs $(i, j)=\left\{\left(p_{1}, L_{o}\right),\left(p_{1}, L_{d}\right),\left(F_{d}, p_{N}\right)\right\}$ and the safe lane change gaps follow from Eqs. (16) to (18) respectively. Regarding communications, the platoon leader $p_{1}$ is responsible for exchanging messages with vehicles in the destination lane. Figure 8 illustrates this strategy and the minimum spacings. The main drawback of this approach becomes visible: the wait for such a big gap leads to road sub-utilization.

### 5.2.2. Leader First

Instead of waiting for a spacing that fits the entire platoon, vehicles can move in one at a time while the gap in the adjacent lane is being generated. Therefore, each platoon vehicle only has to observe safe conditions for itself before starting the maneuver. The platoon leader is again responsible for communications. It monitors distances to the same three vehicles as before: $L_{o}$, $L_{d}$ and $F_{d}$, as illustrated by Figure 9 . For $p_{2}$, determining who is the leader on the current lane and on the destination lane depends on the time $p_{1}$ takes to move in the other lane. If $g_{F_{d}, p_{2}}(t) \geq g_{F_{d}, p_{2}}^{\text {lc }}(t)$


Figure 9: As soon as the platoon leader observes the gaps are safe, it can start lane changing.


Figure 10: If the distance from $F_{d}$ to $p_{2}$ is safe and $p_{2}$ doesn't even "see" $L_{o}$ yet, $p_{2}$ can start lane changing.
is satisfied before $p_{1}$ finishes its maneuver, $p_{2}$ only has to keep track of two distances, as in Fig. 10. The other possibility is that $p_{1}$ finishes the lane change while $p_{2}$ is still waiting. Under normal operation, after $p_{1}$ changes lanes, $g_{p_{2}, L_{o}}(t)>g_{p_{2}, L_{o}}^{\text {lc }}(t)$, but $p_{2}$ must keep track of $L_{o}$ anyway to avoid collisions in case $L_{o}$ has to brake. The relevant gaps are shown in Fig. 11. This reasoning is easily extended for $2<n \leq N$.

The main advantage of this strategy is that, while the platoon waits for the gap to be generated, there is no longer a growing empty region on the road. This improves road utilization. The lane changing vehicles still need to increase the distances to their respective leaders before initiating any lateral movement and to reduce it after finishing the maneuver.

### 5.2.3. Last Vehicle First

In the "Last Vehicle First" strategy, the last platoon vehicle, $p_{N}$, is the first to change lanes. Afterwards, it creates the proper spacing for the others. Therefore, it is now $p_{N}$ who communicates with vehicles in the adjacent lane and it monitors gaps to $p_{N-1}, L_{d}$ and $F_{d}$ to determine when it is safe to start lane changing. Fig. 12 depicts this approach. Concerning the rest of the platoon, vehicle $n$, $n \in\{2, N-1\}$, keeps track of the distances to $n+1, L_{d}$ and $n-1$, which became its follower on the destination lane. Lastly, $p_{1}$ monitors $L_{o}, L_{d}$ and $p_{2}$.

The advantage of this behavior over the previous strategy is that $F_{d}$ only has to actively create space for one vehicle and then it can go back to regular vehicle following behavior. One drawback shows up when the whole platoon must merge into the adjacent lane before a certain point upstream. In this case, the platoon leader, which is the first vehicle to reach the merging point, should also be the first to change lanes.


Figure 11: Vehicle $p_{2}$ has to check its distance to $L_{o}$ to be able to react in case something unexpected happens.


Figure 12: Vehicle $p_{N}$ negotiated the maneuver and starts its lateral movement as soon as gaps are safe.

### 5.3. Numerical Results

As in the experiments of section 3, two vehicle types are used in simulations: passenger vehicles and trucks. Their parameters are detailed in table 1. The safe gaps from $p_{1}$ to $L_{o}$ and $L_{d}$ and from $F_{d}$ to $p_{N}$ are the same as in the single vehicle case. The interest now is how $g_{\text {dest }}\left(F_{d}, L_{d}\right)$ varies with the platoon's velocity, size and type. The parameters $H, \Delta t_{\mathrm{lc}}$, $t_{\mathrm{adj}}$ and the passenger vehicle's maximum deceleration during lane change are all as in table 2 . In addition, a truck's maximum braking during lane change is set to $-2 m / s^{2}$.

Let's first analyze how relative velocities and the number of vehicles in the platoon affect the necessary gap at the adjacent lane. For that, let all vehicles be passenger vehicles and all but the platoon vehicles be traveling at a speed of $90 \mathrm{~km} / \mathrm{h}$. Fig. 13 shows the necessary gaps as a function of relative velocity between lanes for different platoon sizes. Inter-platoon gaps, which influence platoon length, increase quickly with speed because the time headway when preparing for a lane change is larger than the regular vehicle following headway. Therefore performing the maneuver at lower velocity decreases the spacing requirement considerably. However, this also requires a large gap from $F_{d}$ to $p_{N}$ (as in the previous result of Fig. 1) which is not common in congested scenarios.

To see how the spacing requirements change when trucks are involved, a truck platoon is simulated. The number of vehicles in the platoon is fixed at $N=5$ and the surrounding vehicles are again traveling at $90 \mathrm{~km} / \mathrm{h}$. Fig. 14 shows the necessary gaps as a function of relative velocities between lanes for two situations. In blue, the platoon is merging between other two trucks, while, in red, it is merging between two passenger vehicles. Clearly the truck platoons demand much bigger spaces to safely lane change.


Figure 13: Minimum gap at the adjacent lane as a function of the relative velocity between lanes for different number of vehicles in the platoon.


Figure 14: Minimum gap at the adjacent lane for a five vehicle truck platoon as a function of the relative velocity between lanes. In blue, the truck platoon merges between trucks and, in red, it merges between passenger vehicles.

## 6 Validation and Verification of Safety Spacing

This section presents methods and requirements to guarantee that a vehicle can safely determine whether the necessary gaps for lane change have been created. This includes an analysis of the information sources available, a protocol to verify whether measurements are reliable and techniques to identify and assess possible failures.

### 6.1. Information Sources

Once the gap generation procedure starts, the ego-vehicle $E$ must monitor its distances and relative velocities to the three relevant surrounding vehicles: the leader at the origin lane $\left(L_{o}\right)$, the follower at the destination lane $\left(F_{d}\right)$ and the leader at the destination lane $\left(L_{d}\right)$. Lane changing can only start after the safe gaps developed in section 3 are respected. Fig. 15 illustrates the situation and the ego vehicle's sources of information to verify that safety conditions are met. These sources are:

- Sensors: the ego vehicle is assumed to have at least $N$ independent sensors to measure the gap and speed of each of its relevant surrounding vehicles. Example: radar, LiDAR, camera.
- Communications: other vehicles provide some of their parameters, such as mass and maximum deceleration once, and then periodically send $E$ their position, velocity, and acceleration. V2V safety applications are commonly built around Basic Safety Message (BSM) packets, which are sent at a 10 Hz frequency and whose typical communication range is around 500 m [27]. These values fulfill our requirements. Example: DSRC, 5G.
- Dynamical Model: $E$ has a model of its surrounding vehicles and uses it to estimate the value of the next measurement coming from sensors or communication. Example: double integrator with input disturbance.


### 6.2. Verification Protocol

A flowchart of the failure verification procedure for each surrounding vehicle is shown in Fig. 16. This procedure happens in a loop at every $\Delta t$ seconds. The interval length value is defined based on safety standards and computational capacity. All sensors measure the gap and the relative velocity between the ego and the other vehicle. Moreover, through communications, the other vehicle informs its current GPS position, velocity, and acceleration. The first two values allow $E$ to obtain another measurement for the gap and the relative speed. In addition, $E$ 's internal dynamical model of the other vehicle uses the acceleration and velocity acquired through communications to make predictions of the gap and the relative speed. The dashed arrow from Communications to Dynamical Model indicates that the latter uses data provided by the former. All these data sources report their values to a consistency check system. If no inconsistencies are found, the maneuver goes on. Otherwise, a failure identification procedure starts. Once the failure is identified, another system assesses the failure severity. Based on this evaluation, the vehicle can either proceed with the maneuver or abort it and warn the driver.


Figure 15: The ego vehicle monitors its surrounding vehicles. Arrows indicate sensor measurements, an antenna indicates the vehicle is communicating, and the thought bubble represents E's internal model of the others.

### 6.3. Consistency Check and Failure Identification

The protocol described above runs periodically at every $\Delta t$ seconds. At each computation cycle $k$, where $k \in \mathbb{N}$, we use the sensors' and communications' most recent data. The dynamical model, on the other hand, can provide state estimates at any desired time instant. Given the vehicles' accelerations at some time $t^{\prime}<k \Delta t$, the model computes expected values for relative velocity and gap at $k \Delta t$.

To characterize how the consistency check works, let $x_{i}(k)$ be the most recent measured/ estimated value of variable $x$ by source $i$ at the beginning of computation cycle $k$. The possible data origins are $i \in\left\{s_{1}, s_{2}, \ldots, s_{N}, c, m\right\}$, where $s_{n}, c$ and $m$ indicate the information came from sensor $n$, communications or model respectively. All measurements are compared with each other and checked against a threshold. We can express the pairwise comparison results as:

$$
r_{i j}(k)=\left\{\begin{array}{ll}
0, & \text { if }\left|x_{i}(k)-x_{j}(k)\right| \leq \epsilon_{i j}  \tag{25}\\
1, & \text { otherwise } .
\end{array}, \forall j \neq i\right.
$$

where $\epsilon_{i j}$ is a threshold which depends on factors such as sensor noise, communications delay and model accuracy. If $r_{i j}(k)=0, \forall i, j$, there are no inconsistencies and the maneuver can proceed as planned. Otherwise, an inconsistency is detected and the failure identification process is activated. A failure mode $i$ is identified if source $i$ disagrees with half or more of the others. Mathematically:

$$
\begin{equation*}
\sum_{j \neq i} r_{i j}(k) \geq \frac{N+1}{2} \Longrightarrow \text { failure mode } i \text { at cycle } k \tag{26}
\end{equation*}
$$

where $N+2$ is the total number of sources. Given that sensors are independent and with a short enough $\Delta t$, we can assume at most one new failure occurs per cycle.


Figure 16: Flowchart of verification procedure with $N$ independent sensors.

### 6.4. Failure Assessment

Not all failure modes have the same effect on the ego vehicle's ability to verify the gap. Let's study the impact of each one.

### 6.4.1. Sensor Failure Mode

Once a sensor failure is identified, the driver gets a non-emergency warning and that sensor's measurements will be ignored until it is fixed. This failure is considered not risky. To guarantee some redundancy, and therefore safety, we require at least two working sensors at all times. Thus, if a second sensor failure is identified further on, the risk is considered high enough so that the maneuver is aborted and the driver is requested to take control of the vehicle. We define $f_{s_{n}}$ as the failure mode of sensor $n$ and the failure mode for less than two working sensors is called $F_{s}$.


Figure 17: Ego vehicle visibility of vehicles on the destination. On top, all vehicle are visible. At the bottom, a configuration where follower and leader on the origin lane "hide" their counterparts at the destination lane. Red arrows indicate $E$ 's measurements and black arrows show the safe lane change gaps.

### 6.4.2. Communication Failure Modes

We assume the ego vehicle is already communicating periodically with all the surrounding vehicles when a failure is detected. We do this because, if issues in communication arise before the initial contact phase (or if communication is not even established), the gap generation procedure wouldn't have started at all. When the failure mode in communication $f_{c}$ is identified, the ego vehicle will increase its desired safe distance to the other vehicle to account for the loss of reliable information. Moreover, the dynamical model will use the last trustworthy data from the communications system to compute its next estimate. This affects the failure detection thresholds of the consistency check block. Furthermore, if the communication failure is persistent after $K$ cycles, the dynamical model starts using data provided by the sensors.

To evaluate the severity of failure mode $f_{c}$, it is necessary to check whether the other vehicle's states can be measured by the ego vehicle's sensors. While the leader in the origin lane, $L_{o}$, is always visible to $E$, follower and leader in the destination lane, $F_{d}$ and $L_{d}$ respectively, might be shadowed by vehicles in the origin lane. Fig. 17 shows, on top, a situation where both are visible. A configuration where vehicles at the origin lane prevent $E$ 's sensors from obtaining measurements of the vehicles on the destination lane is depicted at the bottom.

The scenario in which $L_{o}$ conceals $L_{d}$ is more likely to happen either when $v_{L_{o}}<v_{L_{d}}$ or when $L_{d}$ has higher braking capability than $L_{o}$, which forces $E$ to keep a bigger gap to its virtual leader than to its real leader. A similar analysis holds true for the case of $F_{o}$ concealing $F_{d}$. In both cases, we have to analyze not only if the vehicle in the destination lane is visible, but also whether the visibility range includes the safe lane change gap. In the illustration at the bottom of Fig. 17, the ego vehicle cannot check whether the safe gap to $L_{d}$ is respected, but it can ascertain that $F_{d}$ is far


Figure 18: Fault Tree for gap verification
away enough. Following this reasoning, we define a conditioning event $c_{v}$ which indicates that $E$ cannot verify the safe gap with it's sensors. Then, failure mode $f_{c}$ is assessed as not risky if the ego vehicle's field of view includes the other vehicle or the safe gap. Otherwise, the system identifies a risky situation and proceeds accordingly.

### 6.4.3. Risk Assessment Tree

Assuming three sensors, Fig. 18 summarizes the behaviors described above. The basic event $f_{i}$ represents failure mode $i \in\left\{s_{1}, s_{2}, s_{3}, c\right\}$ and $c_{v}$ is the conditioning event where $E$ 's sensors cannot check if the safe gap is respected.

## 7 Minimize Impact on Traffic Flow

In this section, we present control strategies that minimize the lane change impact on traffic flow while guaranteeing strict safety constraints. We start by characterizing the several phases of the maneuver, which include longitudinal adjustments before and after the lane change per se. Then, we present the longitudinal control policy that is responsible for: creating safe gaps in the destination lane, increasing gaps to account for loss in braking capability during lateral movement and keeping the lane changing vehicles in the proper longitudinal position. The proposed method is shown to have two important characteristics that minimize impact on traffic. First, whenever a vehicle must increase the distance to its leader, the controller ensures asymptotic convergence to the new gap and velocity setpoint. Secondly, it guarantees string-stability. Simply put, stringstability means that disturbances are attenuated by following vehicles (for a more formal definition
the reader is directed to works such as [40]). Without this characteristic, vehicles upstream would slow down more than vehicles involved in the maneuver, negatively impacting traffic flow and possibly creating stop-and-go behavior. The section ends with the presentation of the lateral controller, responsible for taking the vehicle from one lane to the other once the gaps are safe. This controller also presents asymptotic convergence and is based solely on measurements obtained by the vehicle's sensors.

The techniques presented in this section provide analytical guarantees for the desired behaviors. Impact minimization decisions that rely on simulation results, namely which platoon strategy is best for each scenario, are discussed in section 8 .

### 7.1. Lane Change Phases

When analyzing the lane change impact on traffic, we want to look at the whole process including the adjustments before and after the lane change has taken place. This is particularly important because of our interest in congested scenarios, where there is initially no available safe gap to merge. The different phases of the maneuver are:

0 . Vehicle following: $t \leq t_{0}$
The situation preceding any adjustment towards the lane change is depicted in Fig. 19a. Before $t_{0}$ vehicles perform lane keeping and autonomous vehicle following. By $t=t_{0}$, the ego vehicle $E$, in red, has already performed the necessary communications. Its follower $F_{d}$ and leader $L_{d}$ at the destination lane have been determined and the longitudinal adjustments may start. This phase serves to describe the initial state of the system and is not considered when computing maneuver impacts.

1. Longitudinal adjustments: $t_{0}<t \leq t_{\text {adj }}$

During this phase both lanes are affected by vehicles' longitudinal adjustments. If there is no safe spacing into which $E$ can move, $F_{d}$ starts decelerating to create a big enough gap. By the end of $F_{d}$ 's adjustment it should be at a safe vehicle following distance, $g_{F_{d}, E}\left(t_{\text {adj }}\right) \geq$ $g_{F_{d}, E}^{\text {lc }}\left(t_{\mathrm{adj}}\right)$. Concurrently, $E$ also decelerates to create safe lane changing gaps to its leaders in the original and destination lanes. It is important to remember that gaps for safe lane changing are always larger than gaps for safe vehicle following since the former assumes a decreased braking capability of the ego vehicle. The time $t_{\text {adj }}$ is determined by whichever vehicle, $E$ or $F_{d}$, takes longer to achieve the new desired relative positions. Fig. 19b describes the spacings at this moment.
2. Lateral movement: $t_{\mathrm{adj}}<t \leq t_{\mathrm{lc}}$

This is when the actual lane change occurs. Once all the gaps are safe, the ego vehicle can start turning its front wheels to create lateral movement. By $t=t_{\mathrm{lc}}$, the configuration is as in Fig. 19c. After the lane change, minimum safe following distances are respected.
3. Gap closing: $t_{\mathrm{lc}}<t \leq t_{\mathrm{f}}$

After completing the lane change, vehicle $E$ is farther away from its new leader $L_{d}$ than


Figure 19: Examples of vehicle configurations at the different lane change phases. The red egovehicle must monitor the gaps to the three green vehicles $F_{d}, L_{d}$ and $L_{o}$. The dependence of safe gaps on time is omitted to improve figure clarity.
needed for safe vehicle following. We recall that this is due to the fact that the ego vehicle takes its decreased braking capability into account when computing the lane change gaps. If at the beginning of the maneuver $v_{E}\left(t_{0}\right) \geq v_{L_{d}}\left(t_{0}\right)$, the gap by the end of the lateral movement phase is $g_{E, L_{d}}^{\text {lc }}\left(t_{\text {adj }}\right)$. In this case, $E$ will adjust its position to achieve the minimum safe following gap again, as shown in Fig. 19d. When this happens, at $t=t_{\mathrm{f}}$, the maneuver is completed. Alternatively, if $v_{E}\left(t_{0}\right)<v_{L_{d}}\left(t_{0}\right)$, by the end of the lateral movement phase, $L_{d}$ might be far away from $E$. In this configuration, we consider the maneuver completed at $t_{\mathrm{f}}$ such that $v_{E}\left(t_{\mathrm{f}}\right)=v_{L_{d}}\left(t_{\mathrm{f}}\right)$.

Platoons of vehicles go through the same phases. The main difference is that, depending on the chosen platoon lane changing strategy, different vehicles might be at different phases at the same time instant. We define the platoon's initial and final maneuver times as $t_{0}=\min _{n}\left[t_{0, n}\right]$ and $t_{f}=\max _{n}\left[t_{f, n}\right]$ where the $t_{0, n}$ and $t_{f, n}$ are the initial and final maneuver times for each platoon vehicle $n$. Let's analyze the phases at each of the strategies:

- Synchronous

The only difference to a single vehicle maneuver is that, not only $p_{1}$ must increase its distance to $L_{o}$ but also each platoon vehicle $p_{n}, n>1$, must increase the distance to its respective leader $p_{n-1}$ during the longitudinal adjustments phase. All vehicles start and finish the lateral movement phase together and then proceed to close the gaps to their respective leaders.

## - Leader First

In this case, while a vehicle is still on phase zero, another might have already finished its lateral movement phase. It is important to note that the gap closing phase only starts after all platoon vehicles have finished their lateral movement phase. This implies that, after


Figure 20: In (a), the left lane is moving faster than the right lane. The ego vehicle decided between which vehicles to merge and they can start creating the gap. Afterwards, in (b), $F_{d}$ created the gap between itself and $L_{d}$, but, due to the relative speed between lanes, the gap is not at a suitable location for $E$.
completing their lane change, some vehicles keep constant speed waiting for the rest to finish their own lane changes.

- Last Vehicle First

The analysis is similar to that of the Leader First case but the order in which vehicles enter each phase is reversed.

### 7.2. Gap Generation Controllers

We developed a cooperative decentralized control algorithm that enables the efficient creation of safe lane change gaps in congested scenarios while keeping strict collision avoidance constraints. The approach has the following characteristics: it guarantees that gaps are not only created, but also that they remain available where they are needed, it is easily adapted to different existing vehicle following policies and it requires little more tuning than the vehicle following controller itself. Moreover, it can be directly applied to platoons of vehicles, allowing them to keep their formation after the maneuver is complete.

To achieve that, we studied the most constrained steady-state scenario, where both lanes are at maximum capacity. This implies that all vehicles in the same lane travel at the same speed and the inter vehicle distances are minimum. A solution for this situation can be applied to any less constrained one.

Let's start by noting that, in the analyzed scenario, only $F_{d}$ can generate the proper gap. The difficulty in this phase is one of synchronization: the gap between $F_{d}$ and $L_{d}$ needs to be big enough at a time when $E$ is longitudinally between $F_{d}$ and $L_{d}$. Therefore, merging with non-zero relative speeds would require precise control of several vehicles, which is an unrealistic demand for real-world cases. To see an example of this issue, let $v_{F_{d}}\left(t_{0}\right)>v_{E}\left(t_{0}\right)$ and let's have a moving reference frame fixed on $E$ as in Fig. 20. The gaps on the adjacent lane are moving past $E$ with a positive speed $v_{F_{d}}\left(t_{0}\right)-v_{E}\left(t_{0}\right)$ and $E$ cannot accelerate to "catch-up" since its already following $L_{o}$ at minimum distance. Thus, the gap would have to be created $\left[v_{F_{d}}\left(t_{0}\right)-v_{E}\left(t_{0}\right)\right] t_{g}$ meters upstream of $E$, where $t_{g}$ is the time to generate the gap, and $E$ would have to start its lateral motion precisely when the proper gap is passing by it. Even though we could try to guarantee safety by increasing the minimum gap, which would now also depend on the estimate of $t_{g}$ and communication delays, the main drawback of allowing non-zero relative speeds would still exist:


Figure 21: The solid arrows indicate the real leaders and the dashed ones indicates the virtual leaders.
the risk of the gap "overtaking" $E$. In what follows, we initially provide a controller-agnostic overview of the solution, that is, we describe the method in general lines so that it is not tied to a specific vehicle following control law. Afterwards, the details for the chosen controller are shown as well as the necessary changes to apply this approach to platoons of vehicles.

### 7.2.1. Approach Overview

To avoid the necessity of the previously described fine synchronization, we require that,

$$
\begin{equation*}
v_{F_{d}}(t)=v_{E}(t)=\min \left[v_{L_{o}}(t), v_{L_{d}}(t)\right] . \tag{27}
\end{equation*}
$$

by $t=t_{\text {adj }}$, the time when the lateral movement starts. To achieve this, we assign $L_{d}$ as the virtual leader of $E$ and $E$ as the virtual leader of $F_{d}$. A virtual leader is defined as vehicle which is not in the same lane as the follower, but whose longitudinal position and velocity will be used to determine the follower's acceleration. In Fig. 21 solid arrows indicates the real leaders and the dashed ones indicates the virtual leaders. Under the above requirements, Eqs. (12) and (13) show that the values of $\Delta g_{E, k}, \forall k$ will be zero or negative. Moreover, $E$ can safely keep its speed constant during the lane change, which yields:

$$
\begin{equation*}
g_{i, j}^{\mathrm{lc}}(t) \leq g_{i, j}^{\mathrm{f}}(t), \tag{28}
\end{equation*}
$$

where $(i, j) \in\left\{\left(E_{l c}, L_{o}\right),\left(E_{l c}, L_{d}\right),\left(F_{d}, E\right)\right\}$. Therefore, $E$ (or $F_{d}$ ) has to observe simple safe vehicle following distances to both $L_{o}$ and $L_{d}$ (or $L_{d}$ and $E$ ).

In what follows, we focus our attention on how $F_{d}$ behaves given both its real leader $L_{d}$ and its virtual leader $E$. During the longitudinal adjustments phase, $t_{0}<t \leq t_{\mathrm{adj}}, F_{d}$ computes two control inputs in parallel. The one induced by the real leader is $u_{r}$ and the one induced by the virtual leader is $u_{v}$. The first is computed by whichever vehicle following law is responsible for keeping a safe distance to $L_{d}$. The second input stems from the same control policy, but it is computed based on errors to the virtual leader and it is not safety critical. To see why this distinction is important, one can think of the case where, by $t_{0}$, the gap from $F_{d}$ to $E$ is close to zero, as in Fig. 21. Were the virtual following controller to react as sharply as the regular following controller, $F_{d}$ would brake as hard as possible. This would cause unnecessary great passenger discomfort at $F_{d}$. Therefore, the virtual following controller should have parameters which yield smoother reactions, such as lower maximum braking and jerk. Furthermore, if $F_{d}$ is longitudinally ahead of its virtual

Table 3: Controller Differences

|  | Real Following | Virtual Following |
| :--- | :--- | :--- |
| Compute errors from: | real leader | virtual leader |
| Velocity control if: | leader far ahead | leader far behind |
| Outputs bounded by: | vehicle's dynamics | comfort constraints |

leader, even a moderate deceleration might, with enough time, lead the vehicle to a very low speed. This behavior is again undesired given that the situation is not safety critical. Consequently, the virtual following controller should have a switching logic which keeps the vehicle at a constant minimum speed while $F_{d}$ is still too far ahead of $E$. We note that a similar logic already exists in most regular vehicle following controller. They switch to free flow velocity control when the real leader is too far ahead or if there is no real leader. Table 3 summarizes the main differences between real and virtual vehicle following. Finally, to guarantee safety, after computing inputs from both controllers, $F_{d}$ chooses the minimum - the hardest braking or the smallest acceleration - as its actual input. Thanks to this rule, we know that, at steady state, $F_{d}$ will be at safe distance from $E$ with close to zero relative speed.

One can apply the same reasoning to the ego vehicle following both $L_{o}$ and $L_{d}$. Once $E$ achieves steady-state, it will be at a safe vehicle following distance and at the same velocity of whichever leader is slower and at a larger than necessary distance to the other.

The proposed approach can be applied straightforwardly to most controllers which depend only on the vehicle's immediate preceding vehicle. This includes constant time headway controllers as well as some cited earlier such as [34] and [35]. However, making good use of communications, some laws also rely on the distance between a vehicle and the leader of its platoon [36,40]. In these cases, vehicles can switch from their platoon leader dependent rule to one that relies only on their immediate leaders during the gap generation and lane changing processes.

### 7.2.2. Chosen Vehicle Following Controller

Let the longitudinal dynamics of any vehicle $k$ be modeled as a double integrator:

$$
\left[\begin{array}{c}
\dot{x}_{k}(t)  \tag{29}\\
\dot{v}_{k}(t)
\end{array}\right]=\left[\begin{array}{ll}
0 & 1 \\
0 & 0
\end{array}\right]\left[\begin{array}{c}
x_{k}(t) \\
v_{k}(t)
\end{array}\right]+\left[\begin{array}{l}
0 \\
1
\end{array}\right] a_{k}(t)
$$

and the acceleration be described by:

$$
\begin{equation*}
a_{k}(t)=u_{k}(t)+\Delta_{u_{k}}(t), \tag{30}
\end{equation*}
$$

where $u_{k}$ is the control input and $\Delta_{u_{k}}$ is a disturbance used to model non-linear effects. We opt for a constant time headway policy with headway and velocity errors defined respectively as:

$$
\begin{align*}
& e_{h}(t)=x_{L}(t)-l_{L}-x_{E}(t)-d_{0}-h v(t),  \tag{31}\\
& e_{v}(t)=v_{L}(t)-v_{E}(t), \tag{32}
\end{align*}
$$

where the index $L$ refers to either the real or virtual leader. It should be noted that the headway error is simply the current gap, as defined in Eq. (2), minus the desired gap from Eq. (10). Moreover, the value of the time headway $h$ is chosen following Eq. (9), where parameter $\gamma$ is obtained thanks to V 2 V communication. The control law for vehicle following is:

$$
\begin{equation*}
u_{\text {foll }}(t)=K_{h} e_{h}(t)+K_{v} e_{v}(t)+K_{i} \int_{t_{0}}^{t} e_{h}(\tau) \mathrm{d} \tau \tag{33}
\end{equation*}
$$

One desirable feature of vehicle following controllers is that they are string-stable. Assuming similar vehicles, it can be shown that:

$$
\begin{equation*}
h\left(h K_{p}+2 K_{v}\right) \geq 2 \tag{34}
\end{equation*}
$$

is a necessary and sufficient condition for string-stability. Therefore, the controller gains can be determined by a slightly modified pole placement method that also guarantees string stability.

In case the leader is far ahead, it is assumed that there is no leader and the vehicle switches to a PI controller based on the error to the desired velocity. Similarly, in virtual following, if the virtual leader is too far behind, the controller switches to a PI which keeps the vehicle at its minimum accepted velocity. Moreover, two measures are put in place to produce a smooth behavior at setpoint changes. First, the desired gap goes through a low-pass filter to avoid abrupt control changes. Second, while $E$ increases the distance to its real leader and as long as $g_{E, L_{o}}(t) \geq$ $\alpha g_{E, L_{o}}^{\mathrm{f}}(t)$, where $\alpha \geq 1$ is a design parameter, the maximum braking is limited to a comfortable value.

### 7.2.3. Application to Platoons

When a platoon wants to change lanes, similar steps are taken. In the three strategies, $F_{d}$ adopts $p_{N}$, the last platoon vehicle, as virtual leader and, consequently, creates a gap for the whole platoon. On the other hand, the definition of virtual leader for platoon vehicles depends on the strategy being employed. In the Synchronous case, only $p_{1}$ adds $L_{d}$ as its virtual leader. Since the entire platoon changes lane simultaneously, vehicles $p_{n}, 1<n \leq N$ never have virtual leaders. When the Leader First strategy is used, $p_{1}$ follows the same procedure of a single vehicle. Then, as soon as any vehicle $p_{n}, \forall n$, finishes a lane change, $p_{n+1}$ detects $L_{o}$ as the new real leader and adopts $p_{n}$ as virtual leader. Differently, the Last Vehicle First approach requires a change of virtual leader. When at the origin lane, $p_{n}$ has $p_{n-1}$ as real leader and $L_{d}$ as virtual leader. During the time when $p_{n}$ has completed the lane change phase, but $p_{n-1}$ has not, $p_{n}$ 's real and virtual leaders are $L_{d}$ and $p_{n-1}$ respectively. In other words, $p_{n}$ is $p_{n-1}$ 's follower at the destination lane.

The other particularity of platoon lane changes arises when all platoon vehicles increase their gap setpoints to account for loss of braking capability during lateral movement. If all of them start their longitudinal adjustments simultaneously, an issue related to string-stability is verified. To see this, let the headway error of vehicle $p_{n}$ be:

$$
\begin{equation*}
e_{h, n}(s)=G(s) e_{h, n-1}(s)+\zeta_{n}(s) \tag{35}
\end{equation*}
$$

where $G(s)$ is the transfer function from headway error of preceding vehicle to headway error of following vehicle, and $\zeta_{n}$ models disturbances not caused by $p_{n-1}$. Without loss of generality, let $e_{h, n-1}=0$. When vehicle $p_{n}$ 's setpoint changes, it has to increase its distance to $p_{n-1}$. This causes a disturbance

$$
\begin{equation*}
e_{h, n}(s)=\zeta_{n}(s)=F(s)\left[\left(h_{\mathrm{lc}}-h\right) v_{P}+\left(d_{0, \mathrm{lc}}-d_{0}\right)\right] \tag{36}
\end{equation*}
$$

where $F(s)$ is a low-pass filter to create smooth changes, $v_{P}$ is the platoon velocity, and $h_{\text {lc }}$ and $d_{0, \text { lc }}$ are, respectively, headway and desired distance at standstill computed with lane change braking parameters. Next, note that vehicle $p_{n+1}$ undergoes the same setpoint change $\zeta_{n+1}=\zeta_{n}$. Therefore, its headway error is

$$
\begin{equation*}
e_{n+1, h}(s)=(G(s)+1) \zeta_{n} . \tag{37}
\end{equation*}
$$

Clearly, the disturbance at vehicle $p_{n+1}$ is greater than the one at $p_{n}$. This means that, even though the vehicle following controller is string-stable, if all platoon vehicles adjust their headway setpoint at the same time, vehicles upstream would brake for longer and reduce their speeds to lower values than their leaders. To avoid such an undesired effect, we propose to distribute the disturbances over time: $p_{n}$ only changes its setpoint once vehicle $p_{n-1}$ is back at steady-state. The trade-off is that we change one high impact adjustment for several ones of smaller magnitude.

### 7.3. Lateral Controller

The proposed approach to guarantee maneuver safety first synchronizes vehicles speeds and aligns their positions. Then, while keeping constant velocity, vehicles perform lane change. Therefore at any point in time we have either $\dot{v}_{x} \approx 0$ or $\dot{v}_{y} \approx 0$. This decoupling of longitudinal and lateral movements allows the lateral dynamics to be described by a linear bicycle model:

$$
\frac{\mathrm{d}}{\mathrm{~d} t}\left[\begin{array}{c}
y  \tag{38}\\
v_{y} \\
\theta \\
\dot{\theta}
\end{array}\right]=\left[\begin{array}{cccc}
0 & 1 & 0 & 0 \\
0 & -\frac{a_{1}}{v_{x}} & 0 & -v_{x}-\frac{a_{2}}{v_{x}} \\
0 & 0 & 0 & 1 \\
0 & -\frac{a_{3}}{v_{x}} & 0 & -\frac{a_{4}}{v_{x}}
\end{array}\right]\left[\begin{array}{c}
y \\
v_{y} \\
\theta \\
\dot{\theta}
\end{array}\right]+\left[\begin{array}{c}
0 \\
b_{1} \\
0 \\
b_{2}
\end{array}\right] \delta
$$

where $y, v_{y}$ and $v_{x}$ are,respectively, lateral displacement, lateral velocity and longitudinal velocity based on the vehicle's body coordinates, and $\theta$ is its orientation with respect to the road longitudinal axis. The coefficients are:

$$
\begin{array}{ll}
a_{1}=2 \frac{C_{f}+C_{r}}{m}, \\
a_{2}=2 \frac{C_{f} l_{f}-C_{r} l_{r}}{m}, & b_{1}=2 \frac{C_{f}}{m}, \\
a_{3}=2 \frac{C_{f} l_{f}-C_{r} l_{r}}{I_{z}}, & b_{2}=2 \frac{C_{f} l_{f}}{I_{z}} \\
a_{4}=2 \frac{C_{f} l_{f}^{2}+C_{r} l_{r}^{2}}{I_{z}},
\end{array}
$$



Figure 22: $Y_{r}$ filter to create smooth transition.
where $C_{f}$ and $C_{r}$ are, respectively, front and rear tire cornering stiffness, $l_{f}$ and $l_{r}$ are distances from front and rear tires, respectively, to the center of gravity, $m$ is vehicle's mass, and $I_{z}$ is its yaw moment of inertia. To define the lateral controller, we write out velocities in road coordinates under small angle assumption:

$$
\begin{align*}
& \dot{X}=v_{x} \cos \theta-v_{y} \sin \theta=v_{x}-v_{y} \theta  \tag{39}\\
& \dot{Y}=v_{x} \sin \theta+v_{y} \cos \theta=v_{x} \theta+v_{y}, \tag{40}
\end{align*}
$$

where $X$ and $Y$ are longitudinal and lateral coordinates, respectively, in the road's frame of reference. The final model is:

$$
\begin{equation*}
\dot{q}=A\left(v_{x}\right) q+B \delta \tag{41}
\end{equation*}
$$

where

$$
\begin{aligned}
q & =\left[\begin{array}{llll}
Y & v_{y} & \theta & \dot{\theta}
\end{array}\right]^{T} \\
A\left(v_{x}\right) & =\left[\begin{array}{cccc}
0 & 1 & v_{x} & 0 \\
0 & -\frac{a_{1}}{v_{x}} & 0 & -v_{x}-\frac{a_{2}}{v_{x}} \\
0 & 0 & 0 & 1 \\
0 & -\frac{a_{3}}{v_{x}} & 0 & -\frac{a_{4}}{v_{x}}
\end{array}\right] \\
B & =\left[\begin{array}{llll}
0 & b_{1} & 0 & b_{2}
\end{array}\right]^{T} .
\end{aligned}
$$

Before the lane change $Y\left(t_{0}\right)=Y_{0}$, where $Y_{0}$ is the center of the origin lane. The reference for lane change is $Y_{r}=Y_{0}+\kappa H$, where $\kappa \in\{+1,-1\}$ defines whether the vehicle moves to the left or right lane respectively, and $H$ is the lane's width. Moreover, after the maneuver it is also required that $\left[v_{y}, \theta, \dot{\theta}\right]=[0,0,0]$. The error and its dynamics are therefore

$$
\begin{align*}
e_{y} & =\left[\begin{array}{llll}
Y_{r}-Y & -v_{y} & -\theta & -\dot{\theta}
\end{array}\right]^{T},  \tag{42}\\
\dot{e}_{y} & =A\left(v_{x}\right) e_{y}+B \delta . \tag{43}
\end{align*}
$$

Thus, a state feedback controller $\delta=K e$ can be used to asymptotically track the new reference. To avoid discontinuities in $e$, which would cause large and uncomfortable values of $\delta$, we pass $Y_{r}$ through a low-pass filter as the one used in [42]. It is described by Fig. 22.

Last, we need to address the fact that not all states of $e$ can be measured directly. The lateral position error $Y_{r}-Y$ can be obtained using a camera and the lane markers. The yaw rate $\dot{\theta}$ is directly measured by the vehicle's gyroscope. Using these two measurements, we can use an
observer to estimate the other states.

## 8 Simulation Testing and Evaluation

In this section, we use simulation tools to analyze the traffic impacts of the proposed methods. First, we make use of Simulink to verify the proper behavior of the longitudinal and lateral controllers presented in section 7 . These simulations also provide metrics to compare the platoon lane change strategies efficiency. Then, to evaluate the traffic impact on the road network and on the environment, we present results obtained using the microscopic simulation software VISSIM.

### 8.1. Vehicle Level Simulations

To verify controller behavior and estimate its impacts, we use the simulation software Simulink. It provides a vehicle model block and allows detailed implementation of the control algorithms. We use these capabilities to simulate connected autonomous vehicles and to create the hardest-tomerge scenario. In it, both lanes are assumed to be at full capacity. This entails that vehicles on the same lane have the same initial speed, $v_{o}(0)$ at the origin lane and $v_{d}(0)$ at the destination lane. It also implies that all vehicles are keeping minimum vehicle following distances to their respective leaders until $t_{0}$. Moreover, in all analyzed scenarios:

- There are a three-vehicle platoon and four surrounding vehicles;
- All vehicles are passenger vehicles with parameters defined in table 1 ;
- The longitudinal adjustments phase starts at $t_{0}=5 \mathrm{~s}$ and, at this time instant, the vehicle negotiating the lane change is one meter closer to its virtual leader than to its real leader. Mathematically, $g_{p_{n}, L_{d}}\left(t_{0}\right)=g_{p_{n}, L_{o}}\left(t_{0}\right)-1$, where $n=1$ for strategies Synchronous and Leader First, and $n=3$ for strategy Last Vehicle First.
The initial relative velocity between lanes is defined as:

$$
\begin{equation*}
\Delta v_{\ell}=v_{d}(0)-v_{o}(0) \tag{44}
\end{equation*}
$$

We analyze a total of nine scenarios. For each of the three platoon lane changing strategies, there are three possible initial relative velocities $\Delta v_{\ell}=0, \Delta v_{\ell}=-14.4 \mathrm{~km} / \mathrm{h}$ and $\Delta v_{\ell}=14.4 \mathrm{~km} / \mathrm{h}$. We first present the vehicles' trajectories and errors to verify the platoon lane changing strategies work as intended and to confirm they comply with the safety criteria. Next, the efficiency metrics used to compare the strategies are defined. Last, we the values of these metrics for each strategy.

### 8.1.1. Intended Behavior and Safety

In this section, three scenarios are selected to show the proper lane change phases over time and to confirm the safety criteria is met at all times. Each experiment differs both in lane change strategy and in relative velocity between origin and destination lane.


Figure 23: Vehicle states (a) and relevant errors (b) for platoon lane change with the Synchronous strategy and zero initial relative velocity between lanes. Plots on the left from top to bottom show inter-vehicle gaps, longitudinal velocity and lateral position. Plots on the right from top to bottom show headway, velocity and lateral errors.

The plots in Fig. 23a allow us to verify correct behavior in the case of Synchronous Strategy and $\Delta \mathbf{v}_{\ell}=\mathbf{0}$. The top plot shows the staggered longitudinal adjustment phase. At the start, all platoon vehicles keep the minimum vehicle following distance $g_{i, j}^{\mathrm{f}}$ to their respective leaders. Then, one by one, they increase this distance to $g_{i, j}^{\text {lc }}$. Vehicle $p_{1}$ creates a slightly larger than necessary gap to $L_{o}$ in order to achieve the desired distance to $L_{d}$. The middle plot shows the velocities which cause the gap variation. The lateral movement phase can be identified in the bottom plot. It shows that all vehicles perform the lane change simultaneously, which is the defining characteristic of this strategy. After the lane change, at about $t>55 s$, the gap closing phase starts and platoon vehicles, starting by the last, sequentially decrease the gaps to their leader back to $g_{i, j}^{\mathrm{f}}$. The compliance to safety criteria is better checked in the plots of Fig. 23b. We choose to show error values only up to around $t=t_{\mathrm{lc}}$, when the lateral movement for the entire platoon is completed. After that, real leaders for $p_{1}$ and $F_{d}$ change which make the plots harder to interpret without bringing any gains to the safety analysis. The top two plots present the most relevant headway and velocity errors and the bottom plot shows lateral errors. In this scenario, the critical errors are between $p_{1}$ and its virtual leader $L_{d}$ and between $F_{d}$ and $p_{3}$. The bottom plot confirms that the new reference $Y_{r}$, which causes $e_{y}$ to become positive, is only passed to the vehicles once all headway and velocity errors are non-negative.

The next scenario uses the Leader First strategy and $\Delta \mathbf{v}_{\ell}=-14.4 \mathrm{~km} / \mathrm{h}$, that is, vehicles


Figure 24: Vehicle states (a) and relevant errors (b) for platoon lane change with the Leader First strategy and negative initial relative velocity between lanes. Plots on the left from top to bottom show inter-vehicle gaps, longitudinal velocity and lateral position. Plots on the right from top to bottom show headway, velocity and lateral errors.
are merging into a slower lane. The plots in 24 a confirm the desired behavior for this strategy. The safe lane change gaps are created similarly to the previous case. The difference now is that the platoon has to slow down first, therefore the gap from $p_{1}$ to its leader in the original lane $L_{o}$ grows quickly. It goes off the chart at about $t=10 \mathrm{~s}$. In the velocity plot at the middle, one can check that the platoon synchronized its speed to the adjacent lane. At the bottom plot, the Leader First strategy is clearly visible: vehicles start their lateral movements as soon as they individually arrive at steady-state. We present confirmation of safety criteria for vehicle $p_{1}$ with the plot of relevant errors in Fig. 24b. Again, only the time up to around $t=t_{\mathrm{lc}}$ is show. It is easy to verify that headway and velocity errors (top and middle plots respectively) from $p_{1}$ to $L_{d}$ and from $F_{d}$ to $p_{1}$ are non-negative when the lane change command is given. Vehicle $p_{1}$ 's real leader, $L_{o}$ is not a concern for $p_{1}$ 's safety because it is traveling at a higher speed and thus distances itself quickly. The headway error between $F_{d}$ and $p_{1}$ keeps increasing because $F_{d}$ is still creating a gap for the whole platoon. Errors for the other two platoon vehicles follow similar behavior.

Last, we analyze a scenario with the Last Vehicle First strategy and $\Delta v_{\ell}=14.4 \mathrm{~km} / \mathrm{h}$, that is, the platoon is merging into a faster lane. Starting with the plots in Fig. 25a, the gap plot at the top is quite different from the previous two ones because the last platoon vehicle $p_{3}$ starts the longitudinal adjustments phase first. Another two points to notice in this plot. First, the gap between $p_{3}$ and the leader at the destination lane $L_{d}$ increases quickly and goes off the chart at


Figure 25: Vehicle states (a) and relevant errors (b) for platoon lane change with the Last Vehicle First strategy and positive initial relative velocity between lanes. Plots on the left from top to bottom show inter-vehicle gaps, longitudinal velocity and lateral position. Plots on the right from top to bottom show headway, velocity and lateral errors.
about $t=10 s$ because this vehicle is traveling faster than the platoon. Second, we notice that the gap from $p_{1}$ to $L_{o}$ decreases sharply towards the end of the maneuver. This happens because, once the lane change is completed, platoon vehicles accelerate to match the speed at their new lane, as confirmed by the velocity plot at the middle. The other noticeable difference from this strategy is made evident by the bottom plot, where the lateral movement phases start from last to first platoon vehicle. Maneuver safety regarding $p_{3}$ is confirmed by the relevant error plots in Fig. 25 b . With positive relative velocity between lanes, the virtual leader $L_{d}$ distances itself quickly and is therefore not a concern for $p_{3}$ 's safety. The top and middle plots confirm that headway and velocity errors are positive by the time $p_{3}$ is allowed to start its lateral movement. Errors for $p_{2}$ and $p_{1}$ are similar.

### 8.1.2. Metrics

Before moving on to results on efficiency, we need to define the three metrics used to evaluate it. They are:

## Maneuver Duration

This metric is mathematically defined as:

$$
\begin{equation*}
T=t_{f}-t_{0} \tag{45}
\end{equation*}
$$

## Reserved Space-Time

We need a metric to express road sub-utilization during the maneuver. In [36], maneuver efficiency is measured by the space-time that it occupies, which is defined as the integral of space over time. For our purposes, we want to compute the integral of reserved unoccupied space $r_{k}(t)$ in front of each vehicle $k$. Reserved unoccupied space is defined such that $r_{k}(t)=0$ during regular vehicle following with minimum safe gap:

$$
\begin{equation*}
r_{k}(t)=f_{k}(t)-\left(x_{k}(t)+g_{k, k-1}^{\mathrm{f}}(t)\right) \tag{46}
\end{equation*}
$$

where $f_{k}(t)$ is the longitudinal coordinate of the end of the free space ahead of vehicle $k, x_{k}(t)$ is vehicle $k$ 's front bumper longitudinal coordinate, and $g_{k, k-1}^{\mathrm{f}}(t)$ is the minimum safe vehicle following gap between $k$ and its real leader. In some cases, $f_{k}(t)=x_{k-1}(t)$, that is, the free space is determined by the position of the preceding vehicle. The values of $r_{F_{d}}(t)$ and $r_{p_{2}}(t)$ in Fig. 26a and of $r_{F_{d}}(t)$ in Fig. 26b are computed this way. In other cases, $r_{k}(t)$ depends on the safe lane changing distances. For example, in Fig. 26b, after $p_{1}$ has changed lanes, the gap from $p_{2}$ to $L_{o}$ is bigger than necessary for $p_{2}$ 's lane change. All the additional space greater than $g_{p_{2}, L_{o}}^{\text {lc }}$ ahead of $p_{2}$ is not reserved and thus does not add to $r_{p_{2}}(t)$. A similar situation occurs when one lane is traveling faster than the other. In Fig. 26c, the destination lane's velocity is higher than the origin lane's. While $F_{d}$ is creating a gap for $p_{N}, L_{d}$ is "getting away". Once again, part of the free space between $F_{d}$ and $L_{d}$ is not reserved for the maneuver and is not counted when computing $r_{F_{d}}(t)$. The analysis for the scenario where the origin lane is faster is similar. The final total space-time cost is:

$$
\begin{equation*}
R=\sum_{k=\left\{F_{d}, p_{1}, \ldots, p_{N}\right\}} \int_{t_{0}}^{t_{f}} r_{k}(t) \mathrm{d} t \tag{47}
\end{equation*}
$$

## Acceleration costs

Another interesting metric is the total control effort during the maneuver:

$$
\begin{equation*}
U=\sum_{k=\left\{F_{d}, p_{1}, \ldots, p_{N}\right\}} \int_{t_{0}}^{t_{f}} a_{k}^{2}(t) \mathrm{d} t \tag{48}
\end{equation*}
$$

where $a_{k}$ is the acceleration of vehicle $k$. This metric is usually associated with fuel consumption and passenger comfort.

### 8.1.3. Efficiency

The values of the proposed metrics for each of the nine experiments is presented in table 4. The abbreviations Sync., Ld. F. and L.V.F. on the strategy column stand for Synchronous, Leader First and Last Vehicle First respectively. Regarding maneuver duration and acceleration cost, strategies Synchronous and Leader First present no significant differences. However, when it comes to reserved space-time, the Leader First strategy shows an improvement of at least $20 \%$ for all relative speeds. By allowing vehicles to move in one by one, this approach frees space in the origin lane and occupies the reserved space in the destination lane faster. The Last Vehicle First strategy contrasts


Figure 26: Examples of reserved space $r_{k}(t)$ for different vehicles and in different traffic configurations.

Table 4: Efficiency Results.

| $\Delta v_{L}(k m / h)$ | Strategy | Maneuver <br> Duration $(s)$ | Acceleration <br> Cost $\left(\mathrm{m}^{2} / \mathrm{s}^{3}\right)$ | Reserved <br> Space-Time $(m . s)$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | Sync. | 91 | 359 | 10234 |
|  | Ld. F. | 91 | 367 | 8026 |
|  | L.V.F. | 114 | 321 | 9237 |
|  | Sync. | 86 | 327 | 8089 |
|  | Ld. F. | 85 | 326 | 6388 |
|  | L.V.F. | N/A | N/A | N/A |
|  | Sync. | 85 | 299 | 8826 |
|  | Ld. F. | 85 | 305 | 5750 |
|  | L.V.F. | 98 | 209 | 7810 |

more clearly with the previous two. It is the one which takes the longest to complete, not being able to finish within 2 minutes for the case of negative relative velocity. On the other hand, because it forces $F_{d}$ to decelerate less, this strategy obtains the best acceleration costs. When it comes to reserved space-time, the smaller impact on $F_{d}$ and the longer total maneuver time make this strat-


Figure 27: Reserved space-time costs for each relative velocity and each strategy. The costs are shown separately for $F_{d}$ in blue and for the platoon vehicles in orange.
egy perform somewhere between the other two. To study reserved space-time more deeply, Fig. 27 shows the how much of the cost comes from empty space ahead of $F_{d}$ and how much comes from the platoon. The platoon share includes unoccupied space ahead of $p_{1}$ as well as extra gaps between platoon vehicles. As expected, the Last Vehicle First strategy has the smallest share of reserved space in front of $F_{d}$. The other two strategies have almost the same platoon cost but Leader First has much smaller $F_{d}$ cost.

The optimum lane change trajectory for moving into a slower lane is given by the Leader First strategy. For non-negative relative speeds, the choice depends on the system designer objectives. While the Leader First strategy finishes faster and with considerably lower reserved space-time, the Last Vehicle First yields better acceleration costs. Furthermore, we should remember that $F_{d}$ is the vehicle actively cooperating to generate a gap for others. If one wants to favor a strategy that minimally impacts $F_{d}$, the Last Vehicle First is the best choice.

### 8.2. Microscopic Simulations

To further evaluate the impact of the proposed methods, we use the commercial microsimulation software VISSIM. Whereas the previous experiments in Simulink allowed the detailed control of a small set of vehicles in simplified well-behaved settings, VISSIM supports simulations of thousands of vehicles and their interactions in realistic scenarios. Each vehicle is an agent with built-in vehicle following and lane changing models. At each simulation time step, the vehicle feeds these models with information about its states and its environment to determine its behavior. Vehicle parameters, such as length and maximum speed, as well as driver parameters such as thresholds for increasing or decreasing following distances, can be set by the user. Moreover, VISSIM uses stochastic distributions of speed and spacing thresholds to better depict the variability of human behavior.

Thanks to these capabilities, it is possible to extract commonly used traffic measurements from the simulations. We choose to evaluate the proposed methodologies impacts on traffic flow by two
metrics. To assess our method's efficiency, we use mean travel time:

$$
\begin{equation*}
\bar{T}_{t}=\frac{1}{N} \sum_{n=1}^{N} t_{\mathrm{out}, n}-t_{\mathrm{in}, n} \tag{49}
\end{equation*}
$$

where $N$ is the total number of vehicles that pass a certain region of the highway and $t_{\text {out }, n}$ and $t_{\mathrm{in}, n}$ represent the time instants vehicle $n$ exits and enters the simulation respectively. To estimate the method's impact on safety, we use mean number of stops

$$
\begin{equation*}
\bar{s}=\frac{1}{N} \sum_{n=1}^{N} s_{n} \tag{50}
\end{equation*}
$$

where $s_{n}$ counts how many times vehicle $n$ stopped. Lower values of $\bar{s}$ are associated with smoother traffic flow and smaller crash probability.

The impacts on the environment are gauged by average fuel consumption rate and average $\mathrm{CO}_{2}$, $\mathrm{NO}_{\mathrm{x}}$ and PM25 emission rates over distance traveled. Mathematically,

$$
\begin{equation*}
R=\frac{\sum_{n=1}^{N} E_{n}}{\sum_{n=1}^{N} d_{n}} \tag{51}
\end{equation*}
$$

where $E_{n}$ stands for vehicle $n$ 's fuel consumption or emission of any of the pollutants and $d_{n}$ is the distance traveled by the same vehicle. The values of $E_{n}$ are estimated by the MOVES model. From the Environment Protection Agency's (EPA) website, "EPA's MOtor Vehicle Emission Simulator (MOVES) is a state-of-the-science emission modeling system that estimates emissions for mobile sources at the national, county, and project level for criteria air pollutants, greenhouse gases, and air toxics".

### 8.2.1. Set-up

Since VISSIM creates realistic traffic situations, it is not possible to reproduce the exact full occupancy conditions of previous Simulink simulations. Therefore, we must first describe how to create a relevant scenario in VISSIM.

The experiments take place on a 16 km -long south-bound stretch of the I-710 California highway as shown in Fig. 28. A VISSIM model of this segment was created and calibrated using historical data. To evaluate the effectiveness of the proposed methods, we want a scenario where lane changes occur often in a vehicle-dense environment. To obtain these conditions, an incident that closes the middle lane of a segment with three lanes is created at the location marked by a red dot in Fig. 28. With this, the maximum theoretical vehicle capacity drops from around $8400 \mathrm{veh} / \mathrm{h}$ to around $5600 \mathrm{veh} / \mathrm{h}$. Moreover, to force a high number of lane changes, the simulated vehicle demand is set to be higher than the bottleneck capacity.

The next step is to make vehicles in the simulation behave according to the proposed control algorithms. VISSIM's built-in driver model does not account for connectivity, which means they


Figure 28: I-710 highway segment used for experiments.
cannot follow the communication protocol for gap generation or travel close together as platoons. Even though these limitations originate from the same issue, they are addressed separately. To emulate the behavior of the gap generating controllers, two VISSIM features are used together: lane change recommendation and speed limits. The lane change recommendation sign is created about a kilometer upstream the incident location. This serves two objectives. First, it allows vehicles to know beforehand about the lane closure ahead (as would happen in a connected environment) giving them time to look for appropriate naturally occurring lane change gaps on adjacent lanes. Since the simulation parameters create a dense vehicle area, such spacings are not easily found. Thus, the recommendation upstream of the accident also provides the equivalent necessary time for vehicles to communicate requesting gaps. Then, the speed limits, which are also added to sections upstream the incident, play the role of mimicking the gap generation controller. These limits are given to vehicles at approximately every 500 m interval starting about 5 km upstream of the accident. Moreover, two consecutive limits do not differ by more than $5 \mathrm{mi} / \mathrm{h}$ to allow smooth adaptation and their lower and upper boundaries are $10 \mathrm{mi} / \mathrm{h}$ and $65 \mathrm{mi} / \mathrm{h}$ respectively. This produces the effect of vehicles in the destination lane slowing down to create spacings for those which intend to change lanes. The approach to consider the effect of platoons is to include trucks in the simulation. Since trucks are longer, they consume more fuel and require larger gaps to merge, as do platoons.

### 8.2.2. Road-Level Results

For the results presented here, the lane closure only starts after 5 minutes of simulation. This avoids corrupting the measurements with transient behavior at start-up. In addition, the incident remains in place until the simulation ends, which takes another 40 minutes. The chosen demands are 6000 $\mathrm{veh} / \mathrm{h}$ and $6500 \mathrm{veh} / \mathrm{h}$ with $15 \%$ of platoons. To decrease sensibility due to the randomness of VISSIM simulations, all presented values are the average of 10 sets of Monte Carlo runs with different seeds. The results presented in Table 5 show the advantages of the proposed approach. Without any control, the bottleneck with high vehicle density creates the inefficient and unsafe situation where a vehicle stuck behind the closure has close to zero velocity and must merge into

Table 5: VISSIM Simulation Results.

|  | $6000 \mathrm{veh} / \mathrm{h}$ |  |  | $6500 \mathrm{veh} / \mathrm{h}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No Control | Control | Improvement | No Control | Control | Improvement |
| $\bar{T}_{t}(s)$ | 19.84 | 16.69 | $-15.89 \%$ | 21.25 | 16.55 | $-22.13 \%$ |
| $\bar{s}$ | 15.46 | 1.74 | $-88.75 \%$ | 16.12 | 1.83 | $-88.65 \%$ |
| Energy $(\mathrm{kJ} / \mathrm{mi})$ | 182.55 | 169.39 | $-7.21 \%$ | 182.85 | 168.11 | $-8.06 \%$ |
| $\mathrm{CO}_{2}(\mathrm{~g} / \mathrm{mi})$ | 570.72 | 529.76 | $-7.18 \%$ | 568.96 | 523.25 | $-8.04 \%$ |
| $\mathrm{NO}_{\mathrm{x}}(\mathrm{g} / \mathrm{mi})$ | 1.58 | 1.50 | $-4.95 \%$ | 1.58 | 1.50 | $-4.95 \%$ |
| PM25 $\left.^{2} \mathrm{~g} / \mathrm{mi}\right)$ | 0.052 | 0.050 | $-3.74 \%$ | 0.052 | 0.050 | $-3.74 \%$ |

a fast speed adjacent lane. This causes the lane changing vehicle to wait considerably for a proper gap and creates a queue on that lane. Furthermore, it forces vehicles in the destination lane to slow significantly and in a short time interval to avoid accidents when lane changes take place in front of them. Such a behavior yields an elevated number of stops. Not surprisingly, the value $\bar{s}$ presents the greatest decrease once control is applied. Not only that, thanks to a smoother traffic flow, the total travel time $\bar{T}_{t}$ also decreases substantially. Improving these two metrics has a direct effect on the environmental impact. The relative gains in fuel consumption and $\mathrm{CO}_{2}$ emissions are nearly the same since carbon dioxide is the main byproduct of fuel combustion. The other two measured pollutant emissions are also lowered.

### 8.3. Lane-Level Results

To deepen our understanding of the controller effects on the scenario, we analyze what happens on each lane as a function of the vehicle demand. The variables that best describe the state of traffic flow on a lane are vehicle density and flow-rate. The first, henceforth called $\rho$, is measured in vehicles per mile (per lane) and it represents how tightly packed are vehicles on a certain segment of the lane. The second, $q$ from here on, measures how many vehicles pass by that lane segment within a time interval and its typical unit is vehicles per hour (per lane). The relationship between these variables has been shown to follow what is commonly known as the triangular fundamental diagram [43]. Fig. 29 illustrates what a typical fundamental diagram looks like for a road section. While this shape is obtained from rigorous math, for our purposes we can proceed with an intuitive understanding of it. The first part of the diagram, with positive slope, represents an uncongested situation, where an increase in density means an increase in flow. However, roads have limited capacity, denoted $C$ in the diagram. After the corresponding critical density $\rho^{c}$, the road is in a congested state, where higher densities imply lower flows. One can easily visualize this in a busy highway during rush-hour: when vehicles are all bumper to bumper, the density is elevated but the flow is considerably low.

To evaluate the effects of the lane change controller, the same previously described scenario is used, but now traffic demand is varied over a wide range of values. It is useful to note that the capacity per lane is estimated at around 2800 veh $/ h$ before the incident happens. Density


Figure 29: Example of typical fundamental diagram.


Figure 30: Fundamental diagrams with (blue) and without (red) lane change control for the remaining open lanes.
and flow at the open lanes (first and third) at the segment immediately upstream of the incident are measured in simulations both with and without control. Results are shown in Fig. 30, where red squares indicate the measurements without control and blue circles the ones with control. It's visible that both lanes are equally affected by the closure of the middle lane and they achieve a maximum flow of approximately $2400 \mathrm{veh} / h$ in the case without control. This is due to the previously explained behavior of slow vehicles having to force themselves in the faster adjacent lanes. On the other hand, the flow goes up to around $2800 \mathrm{veh} / h$ in the scenario with controller showing that maximum capacity is again achieved despite the incident. We conclude that the proposed methods can increase traffic flow efficiency.

## 9 Conclusion

In this work, we studied several challenges concerning safe autonomous lane changes in congested scenarios. Our solution to address safety and efficiency simultaneously is based on vehicle communication and cooperation. The first step was to define what constitutes a safe gap for lane changing. Besides relying on previous results of safe vehicle following and lane change kinematics, we make a conservative assumption to define a constant maximum braking during lane change. The main advantage of this method is seen later on when this larger spacing request is seamlessly included in the longitudinal vehicle controller. Concerning communication, we proposed a protocol that details what information vehicles should exchange and how to avoid conflicts. After reviewing works on the DSRC standard and its performance, we conclude that our protocol can be implemented. We also analyze how to expand safety spacing requirements and communications to platoons of vehicles. Our solution is based on three strategies, Synchronous, Leader First and Last Vehicle First, which can ensure the platoon remains together after the lane change. Next, we examined how to make use of physical and analytical redundancies to verify the requested gaps are indeed created. The proposed methodology compares several measurements and internal model predictions to check for consistency and identify possible system faults. It also defines the severity of each failure and when the driver is asked to take control. Then, we developed control policies that enable lane changes in the most congested scenarios while satisfying the strict safety requirements. Thanks to communications, vehicles with the intention to change lanes as well as the one responsible for creating the gap can adopt a virtual leader. They keep using the preceding vehicle on their own lanes as the real leader for safety purposes while the virtual leader makes them longitudinally adjust themselves for the maneuver. We showed how this can be achieved with two parallel constant time headway controllers through simulations. Moreover, these experiments allowed us to compare the efficiency of the three proposed platoon strategies. We establish that the Leader First strategy is, on average, the best. However, the Last Vehicle First option is advantageous if one wants to minimize the discomfort of the vehicle creating the gap. After confirming the expected behavior from controllers, VISSIM simulations are used to assess impacts on traffic flow and the environment. The microscopic simulation scenario set-up is explained and its results indicate that the developed methods improve both safety and efficiency. With no control, vehicles queue waiting for a proper lane change space and, when they find one, they force large decelerations on the destination lane. With control, the traffic flow gets smoother since vehicles are able to perform lane changes before being forced to a full stop. This decreases the number of stops and the total travel time. Consequentially, fuel consumption and pollutant emission rates are also lowered. The gains in efficiency are corroborated by a deeper study of individual lanes' densities and flows. The fundamental diagrams show improved traffic flow in the simulations with control.

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## 11 Data Management Plan

## Products of Research

This study did not collect any data. All results are obtained from simulations whose parameters are described in the report.
Data Format and Content
Not applicable.
Data Access and Sharing
Further details about the results obtained in this study can be obtained by contacting the authors at fvallada@usc.edu.

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